Properties of generating functions

nth term of the sequence	sequence	generating function	compact form of gf
g_n	$(g_0, g_1, g_2, g_3, \ldots)$	$\sum_{n=0}^{\infty} g_n s^n$	$G(s)$
$\alpha g_n + \beta f_n$	$(\alpha g_0 + \beta f_0, \alpha g_1 + \beta f_1, \ldots)$	$\sum_{n=0}^{\infty} (\alpha g_n + \beta f_n) s^n$	$\alpha G(s) + \beta F(s)$
g_{n+1}	$(g_1, g_2, g_3, g_4, \ldots)$	$\sum_{n=0}^{\infty} g_{n+1} s^n$	$\frac{1}{s}\Big[G(s) - g_0\Big]$
g_{n+2}	$(g_2, g_3, g_4, g_5, \ldots)$	$\sum_{n=0}^{\infty} g_{n+2} s^n$	$\frac{1}{s^2}\Big[G(s) - g_0 - g_1 s\Big]$
$g_{n+k},\ k \in \mathbf{N}$	$(g_k, g_{k+1}, g_{k+2}, g_{k+3}, \ldots)$	$\sum_{n=0}^{\infty} g_{n+k} s^n$	$\frac{1}{s^k}\Big[G(s) - \sum_{i=0}^{k-1} g_i s^i\Big]$
$g_{n-1}\mathbf{1}(n)$	$(g_{-1}, g_0, g_1, g_2, \ldots)$	$\sum_{n=0}^{\infty} g_{n-1} s^n$	$g_{-1} + sG(s)$
$g_{n-2}\mathbf{1}(n)$	$(g_{-2}, g_{-1}, g_0, g_1, \ldots)$	$\sum_{n=0}^{\infty} g_{n-2} s^n$	$g_{-2} + g_{-1}s + s^2 G(s)$
$g_{n-k}\mathbf{1}(n),\ k \in \mathbf{N}$	$(g_{-k}, g_{-k+1}, g_{-k+2}, \ldots)$	$\sum_{n=0}^{\infty} g_{n-k} s^n$	$s^k G(s) + \sum_{i=0}^{k-1} g_{-k+i} s^i$
$g_{n-1}\mathbf{1}(n-1)$	$(0, g_0, g_1, g_2, \ldots)$	$\sum_{n=1}^{\infty} g_{n-1} s^n$	$sG(s)$
$g_{n-2}\mathbf{1}(n-2)$	$(0, 0, g_0, g_1, \ldots)$	$\sum_{n=2}^{\infty} g_{n-2} s^n$	$s^2 G(s)$
$g_{n-k}\mathbf{1}(n-k),\ k \in \mathbf{N}$	$(0, 0, \ldots, g_0, g_1, \ldots)$	$\sum_{n=k}^{\infty} g_{n-k} s^n$	$s^k G(s)$
$a^n g_n$	$(g_0, a g_1, a^2 g_2, a^3, g_3, \ldots)$	$\sum_{n=0}^{\infty} a^n g_n s^n$	$G(as)$
$(n+1) g_{n+1}$	$(g_1, 2g_2, 3g_3, 4g_4, \ldots)$	$\sum_{n=0}^{\infty} (n+1) g_{n+1} s^n$	$G'(s)$
$n g_n$	$(0, g_1, 2g_2, 3g_3, \ldots)$	$\sum_{n=0}^{\infty} n g_n s^n$	$sG'(s)$
$(n+1)(n+2) g_{n+2}$	$(2g_2, 2 \cdot 3 g_3, 3 \cdot 4 g_4, \ldots)$	$\sum_{n=0}^{\infty} (n+1)(n+2) g_{n+2} s^n$	$G''(s)$
$n^2 g_n$	$(0, g_1, 4g_2, 9g_3, \ldots)$	$\sum_{n=0}^{\infty} n^2 g_n s^n$	$s^2 G''(s) + sG'(s)$
$\frac{1}{n} g_{n-1}\mathbf{1}(n)$	$(0, g_0, \frac{g_1}{2}, \frac{g_2}{3}, \ldots)$	$\sum_{n=1}^{\infty} \frac{1}{n} g_{n-1} s^n$	$\int_0^s G(t)\,dt$
$f_n * g_n$	$(f_0 g_0, f_0 g_1 + f_1 g_0, \ldots)$	$\sum_{n=0}^{\infty} \left(\sum_{k=0}^{n} f_k g_{n-k} \right) s^n$	$F(s)G(s)$
$\sum_{k=0}^{n} g_k$	$(g_0, g_0 + g_1, g_0 + g_1 + g_2, \ldots)$	$\sum_{n=0}^{\infty} \left(\sum_{k=0}^{n} g_k \right) s^n$	$\frac{1}{1-s} G(s)$

Generating functions of some sequences

nth term of the sequence	sequence	generating function	compact form of gf
$\mathbf{1}(n)$	$(1, 1, 1, \ldots)$	$\sum_{n=0}^{\infty} s^n$	$\frac{1}{1-s}$
$\mathbf{1}(n-k)$	$(1, 1, 1, \ldots)$	$\sum_{n=k}^{\infty} s^n$	$\frac{s^k}{1-s}$
a^n	$(1, a, a^2, a^3, \ldots)$	$\sum_{n=0}^{\infty} a^n s^n$	$\frac{1}{1-as}$
$(-1)^n$	$(1, -1, 1, -1, \ldots)$	$\sum_{n=0}^{\infty} (-1)^n s^n$	$\frac{1}{1+s}$
n	$(0, 1, 2, 3, \ldots)$	$\sum_{n=0}^{\infty} n s^n$	$\frac{s}{(1-s)^2}$
$n+1$	$(1, 2, 3, 4, \ldots)$	$\sum_{n=0}^{\infty} (n+1) s^n$	$\frac{1}{(1-s)^2}$
$(n+1)a^n$	$(1, 2a, 3a^2, 4a^3, \ldots)$	$\sum_{n=0}^{\infty} (n+1) a^n s^n$	$\frac{1}{(1-as)^2}$
$\delta(n)$	$(1, 0, 0, 0, \ldots)$	$\sum_{n=0}^{\infty} \delta(n) s^n$	1
$\delta(n-1)$	$(0, 1, 0, 0, \ldots)$	$\sum_{n=0}^{\infty} \delta(n-1) s^n$	s
$\delta(n-k),\ k \in \mathbf{N}$	$(0, 0, \ldots, 0, 1, 0, \ldots)$	$\sum_{n=0}^{\infty} \delta(n-k) s^n$	s^k
$\sin n\varphi$	$(0, \sin\varphi, \sin 2\varphi, \sin 3\varphi, \ldots)$	$\sum_{n=0}^{\infty} \sin n\varphi s^n$	$\frac{s \sin\varphi}{1 - 2s\cos\varphi + s^2}$
$\cos n\varphi$	$(0, \cos\varphi, \cos 2\varphi, \cos 3\varphi, \ldots)$	$\sum_{n=0}^{\infty} \cos n\varphi s^n$	$\frac{1 - s\cos\varphi}{1 - 2s\cos\varphi + s^2}$
$\sin n\frac{\pi}{2}$	$(0, 1, 0, -1, 0, 1, \ldots)$	$\sum_{n=0}^{\infty} s^n$	$\frac{s}{1+s^2}$
$\cos n\frac{\pi}{2}$	$(1, 0, -1, 0, 1, 0, \ldots)$	$\sum_{n=0}^{\infty} s^n$	$\frac{1}{1+s^2}$
$\frac{a^n}{n!}$	$(1, \frac{a}{1!}, \frac{a^2}{2!}, \frac{a^3}{3!}, \ldots)$	$\sum_{n=0}^{\infty} \frac{a^n}{n!}, s^n$	e^{as}
$\binom{c+n-1}{n}$	$(1, c, \binom{c+1}{2}, \binom{c+2}{3}, \ldots)$	$\sum_{n=0}^{\infty} \binom{c+n-1}{n} s^n$	$\frac{1}{(1-s)^c}$
$\frac{1}{n}\mathbf{1}(n)$	$(0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots)$	$\sum_{n=1}^{\infty} \frac{1}{n} s^n$	$-\ln(1-s)$
$\frac{(-1)^{n+1}}{n}\mathbf{1}(n)$	$(0, 1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \ldots)$	$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} s^n$	$\ln(1+s)$