INTEGRALS

I Basic rules (assuming that functions are integrable on some interval I; $C \in \mathbb{R}$)

1. Definition: $\int f(x)\,dx = F(x) + C$ where $F'(x) = f(x)$ for any $x \in I$.

2. The integral of constant multiple is equal to the constant multiple of integral

$$\int k f(x)\,dx = k \int f(x)\,dx, \quad k \in \mathbb{R}.$$

3. The integral of sum is equal to the sum of integrals

$$\int f(x) + g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx.$$

4. Integration by parts $(u, v \in C^1(I))$

$$\int u(x)v'(x)\,dx = u(x)v(x) - \int u'(x)v(x)\,dx.$$

5. Substitution Rule

$$\int f[g(x)]g'(x)\,dx = \left[\begin{array}{c} g(x) = t \\ g'(x)\,dx = dt \end{array} \right] = \int f(t)\,dt = F(t) + C = F[g(x)] + C.$$

6. Useful formula $(f \in C^1(I),\ f(x) \neq 0 \text{ for } x \in I)$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C.$$

II Integrals of basic functions

1. $\displaystyle\int dx = x + C$

2. $\displaystyle\int x\,dx = \frac{1}{2}x^2 + C$

3. $\displaystyle\int x^s\,dx = \frac{1}{s+1}x^{s+1} + C, \quad s \neq -1$

4. $\displaystyle\int \frac{dx}{x} = \ln|x| + C$

5. $\displaystyle\int \frac{dx}{\sqrt{x}} = 2\sqrt{x} + C$

6. $\displaystyle\int \frac{dx}{x^2} = -\frac{1}{x} + C$

7. $\displaystyle\int e^x\,dx = e^x + C$

8. $\displaystyle\int a^x\,dx = \frac{a^x}{\ln a} + C, \quad a > 0, a \neq 1$

9. $\displaystyle\int \sin x\,dx = -\cos x + C$

10. $\displaystyle\int \cos x\,dx = \sin x + C$

11. $\displaystyle\int \frac{dx}{\sin^2 x} = -\cot x + C$

12. $\displaystyle\int \frac{dx}{\cos^2 x} = \tan x + C$

13. $\displaystyle\int \frac{dx}{x^2 + 1} = \arctan x + C$

14. $\displaystyle\int \frac{dx}{\sqrt{1 - x^2}} = \arcsin x + C$

15. $\displaystyle\int \sinh x\,dx = \cosh x + C$

16. $\displaystyle\int \cosh x\,dx = \sinh x + C$

17. $\displaystyle\int \frac{dx}{\sinh^2 x} = -\coth x + C$

18. $\displaystyle\int \frac{dx}{\cosh^2 x} = \tanh x + C$