name	surname	group	date	*points*	add.p.	*sum*	*exam*	classes	*fin. mark*

Test in Calculus — Macrocourse sem. II — version B0

1. Function g is continuous on $\langle 0,10\rangle$ and $f(x)=\int_x^8 g(t)\,dt$. If $f(0)=1$ and $f(5)=-12$, then $\int_1^5 g(t)\,dt=$ ....

2. Function g is even and integrable over $\langle -10,10\rangle$. If $\int_{-6}^0 g(t)\;dt=-4$, then $\int_{-6}^6 g(t)\;dt=$ ....

3. Function f is odd and continuous on **R**. If $\int_{-2}^5 f(x)dx=12$, then $\int_2^5 f(x)dx=$ ....

4. Function f is odd and continuous on **R**. If $\int_{-8}^3 f(x)dx=10$ and $\int_5^8 f(x)dx=-3$, then $\int_3^5 f(x)dx=$ ....

5. The mean value of function $f(x)=\sin x$ (calculated from the Mean-Value Theorem for definite integral) is negative in intervals:

A. $\langle 0,\pi\rangle$ **B.** $\langle -\frac{\pi}{3},\frac{\pi}{6}\rangle$ **C.** $\langle 0,\frac{3\pi}{2}\rangle$ **D.** $\langle \frac{-\pi}{6},\frac{2\pi}{3}\rangle$

6. Let $f(x,y)=\int_x^y \frac{\sin\sqrt{t}}{t-2}\,dt$. Then $\frac{\partial f}{\partial x}=$ .... and $\frac{\partial f}{\partial y}=$ ....

for (x,y) such that ....

7. Improper integrals are

A. $\int_1^{+\infty} \arccos\frac{1}{x}\,dx$ **C.** $\int_0^1 \frac{dx}{\arccos x}$ **E.** $\int_{-\infty}^1 \arctan x$

B. $\int_{-1}^0 \frac{dx}{\arccos x}$ **D.** $\int_0^{+\infty} \arccos(4x^2-1)\;dx$ **F.** $\int_{-\infty}^{+\infty} \sqrt{\text{arccot}\,(x-1)}dx$

8. Initial conditions for the equation $2y''-3y'+\ln(y-2)=\arcsin 2x$ may be, for example, following:

....

9. A particular solution of $y''-y'=x^2+x$ is of the form ....

10. A particular solution of $y''-2y'+17y=e^{-x}\sin 2x$ is of the form ....

11. A particular solution of $y''-2y'+y=e^x$ is of the form ....

12. $\mathcal{L}[(3+2e^{-t}+t\sin\frac{t}{3})\mathbf{1}(t)]=$ ....

13. $\mathcal{L}^{-1}\left[\frac{5}{s^4}+\frac{2s+1}{(s-3)^2+5}\right]=$ ....

14. Complete (functions $y_1,y_2,\ldots,y_n$ are from C^{n-1}–class):

A. If $W(y_1,y_2,\ldots,y_n)=0$, then $y_1,y_2,\ldots,y_n$ are linearly ....

B. If $W(y_1,y_2,\ldots,y_n)\neq 0$, then $y_1,y_2,\ldots,y_n$ are linearly ....

C. If $y_1,y_2,\ldots,y_n$ are linearly independent, then $W(y_1,y_2,\ldots,y_n)$..

D. If $y_1,y_2,\ldots,y_n$ are linearly dependent, then $W(y_1,y_2,\ldots,y_n)$..

15. The partial derivative of $u=f(x,y,z)$ with respect to z at $P(x_0,y_0,z_0)\in D_f$ is defined as

....

if ....

16. The total differential of function $z = f(x, y)$ at $P(a, b) \in D_f$ is equal to ....

iff the increment of function $\Delta z =$..

may be written in the form: $\Delta z =$...

where ....

17. Directional derivative of $z = 3x^2 - 2y^3$ at $P(1, 0)$ in the direction of $\mathbf{u} = [-5, 2]$ equals .... It means that the function in this direction ....

18. Tangent plane to $x^2y - 3y^2z + z^2x = 5x$ at $P(1, 1, -1)$ has equation ....

19. Let $u = \frac{x}{y} + \cos z$ and $P(2, \frac{1}{3}, \pi)$. Then $\operatorname{grad} u|_P$ equals ....

20. Mark each of the following sentences T (if it is true) or F (if it is false). All sentences concern real–valued functions of two real variables.

.... 1) Continuity is necessary for differentiability.
.... 2) Differentiability is necessary for continuity.
.... 3) If a function has both partial derivatives, then it is continuous.
.... 4) If a function has both partial derivatives, then it is differentiable.
.... 5) All functions from C^1–class are differentiable.
.... 6) If a function is differentiable, then it is continuous.
.... 7) If a function is differentiable, then its partials are continuous.

21. If $u = f(x, y, z)$ and $f \in C^3$, then

A. $\frac{\partial^4 u}{\partial x^2 \partial z^2} = \frac{\partial^4 u}{\partial z^2 \partial x^2}$ **C.** $\frac{\partial^3 u}{\partial x^2 \partial y} = \frac{\partial^3 u}{\partial x \partial y \partial x}$ **E.** $\frac{\partial^3 u}{\partial x^2 \partial y} = \frac{\partial^3 u}{\partial y \partial x^2}$

B. $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial z}$ **D.** $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ **F.** $\frac{\partial^3 u}{\partial x^2 \partial y} = \frac{\partial^3 u}{\partial x \partial y^2}$

22. A function f is continuous on $D = \{(x, y) : \ -1 \le x \le 0 \ \wedge x + 1 \le y \le 1 - x^2\}$. Then:

$$\iint_D f(x, y)\, dxdy = \int_{.....}^{.....} \left[\int_{.....}^{.....} f(x, y)\, dy\right] dx = \int_{.....}^{.....} \left[\int_{.....}^{.....} f(x, y)\, dx\right] dy$$

23. A function f is continuous on $D = \{(x, y) : \ x^2 + y^2 \le 5 \ \wedge |y| \ge x\}$. Then:

$$\iint_D f(x, y)\, dxdy = \int_{.....}^{.....} \left[\int_{.....}^{.....} \text{...}\, dr\right] d\varphi$$

24. Complete the definition of double integral:

If $f : \ D \to \mathbf{R}$ (where $D \subset \mathbf{R}^2$ is ....)
is .... on D, then
$\iint_D f(x, y)\, dxdy =$...

iff for all .... always exists and always has the same ....
independently of ....

In this definition:
ΔD_i is ....,
$(\xi_i, \eta_i) \in$ ...., $i =$ .... are ....
$\lambda_n =$ .... is ....