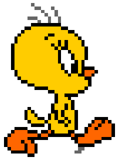
MOTION OF MASS IN SPACE

GENERAL CLASSIFICATION OF MOTION

Motion of a particle in space:

- **forms - domain of kinematics**
- **causes - domain of dynamics**



Description of a motion of particle in space – not so simple !
only possible after basic approximation (simplification)

Main idea: material point (Newton - 1685)

- material object having a mass for which dimension (volume) with respect to space can be neglected !

Motion of particle: motion of material point described by simple mathematical relations (modelling)

Examples:
- **electron in atom like hydrogen H**
- **tennis ball on a court or on a playground**
- **planets in Solar System, Moon in motion around Earth**

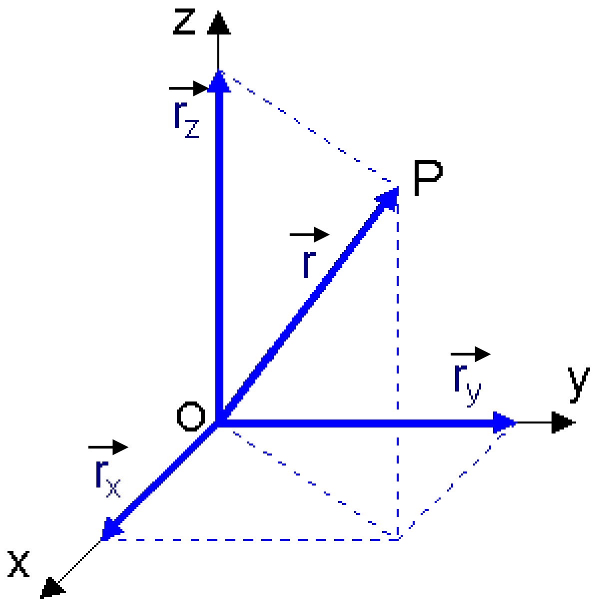
KINEMATICS OF MATERIAL POINT

Motion of particle in space: several main concepts and parameters

POSITION

Location in coordinate reference systems (frames)

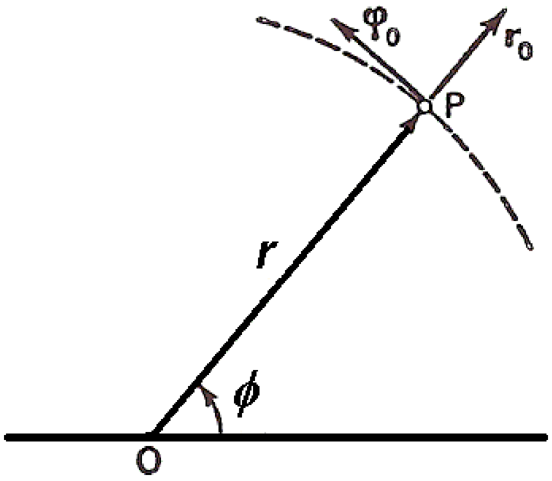
Cartesian



$$\vec{r} = \vec{r}_x + \vec{r}_y + \vec{r}_z$$

$$|\vec{r}| = \sqrt{|\vec{r}_x|^2 + |\vec{r}_y|^2 + |\vec{r}_z|^2}$$

Polar



$$d\vec{r} = \vec{r} \times d\phi$$

KINEMATICS OF MATERIAL POINT

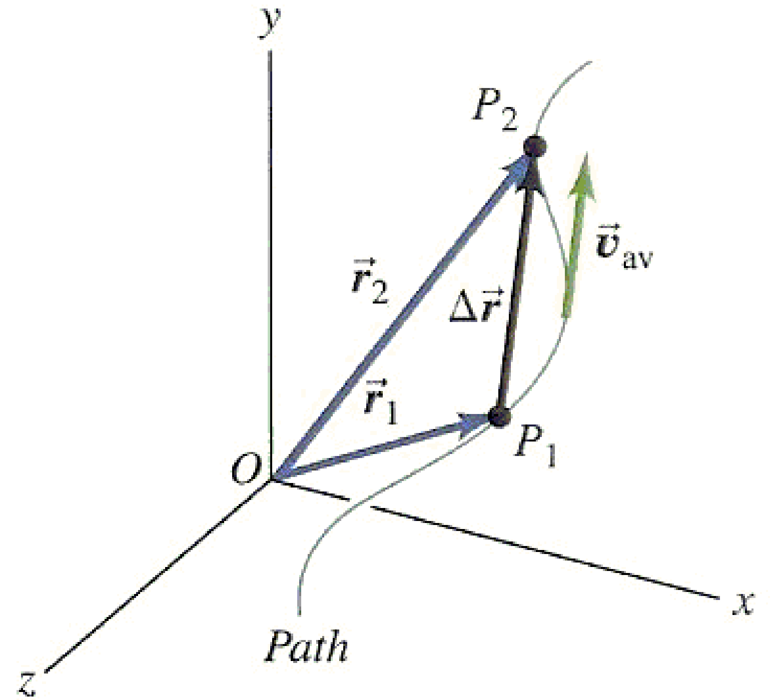
MOTION

Relative change of particle position in coordinate reference system

DISPLACEMENT

Vector of relative change of position in reference system (frame)

- **linear:**



Proper value of the vector - space interval of points 1 and 2

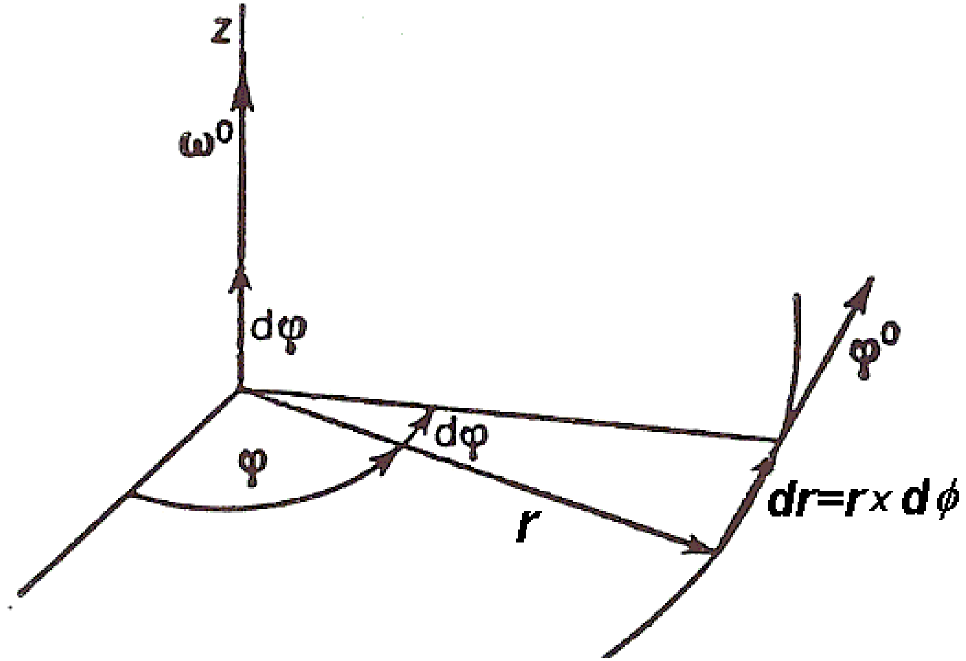
$$r_{12} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

KINEMATICS OF MATERIAL POINT

DISPLACEMENT

Vector of relative change position in specified reference system (frame)

- **angular:**



Proper value of the vector - angular interval

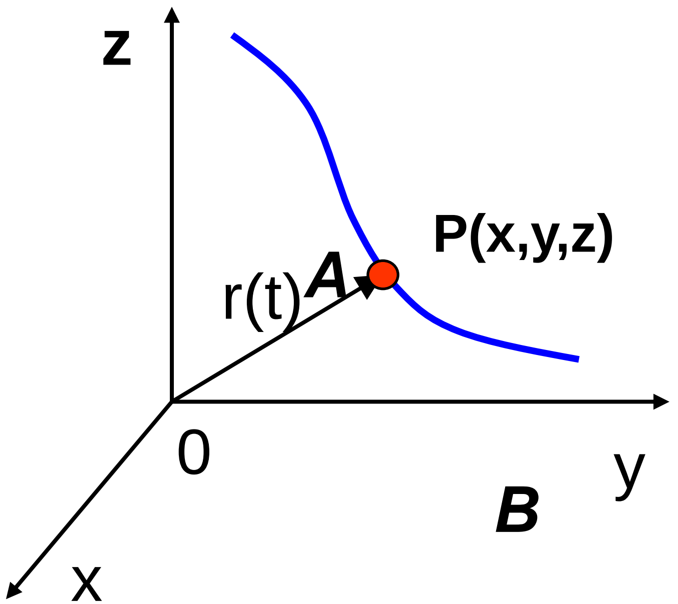
$$d\vec{r} = \vec{r} \times d\phi$$

KINEMATICS OF MATERIAL POINT

TRAJECTORY (PATH)

Line encircled by particle during motion - geometric place (locus) of end of displacement vector

$$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$$



DISTANCE

Interval length of trajectory (path) traversed by a particle between chosen points (A-B)

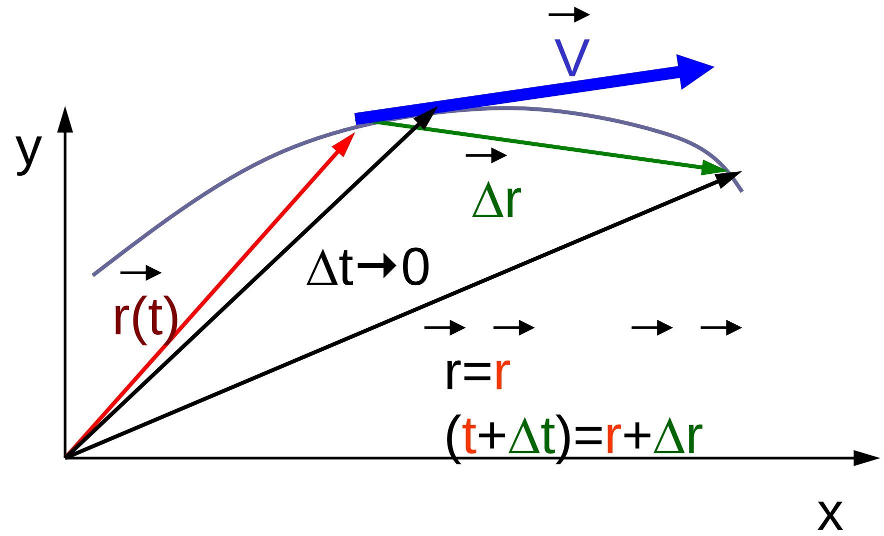
$$S_{AB} = \int_A^B \frac{d\vec{r}}{dt} dt = \int_A^B \vec{\upsilon} \cdot dt$$

KINEMATICS OF MATERIAL POINT

VELOCITY

First derivative of position vs time (instantaneous)

$$\vec{\upsilon} = \frac{d\vec{r}}{dt}$$



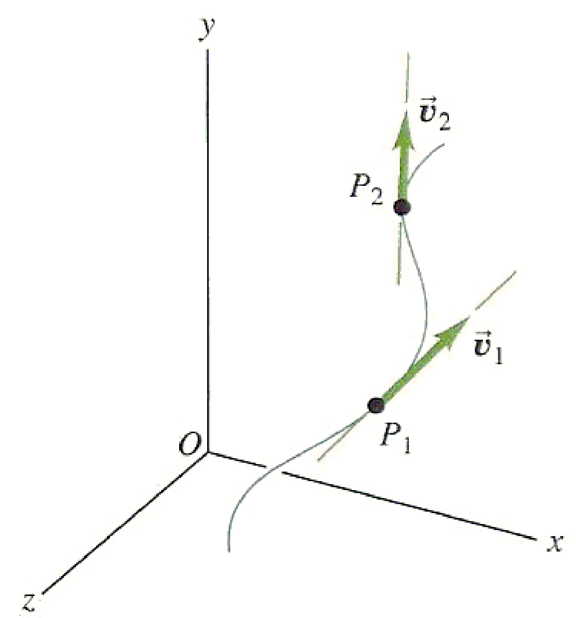
- **linear**

first derivative of position coordinates vs time

$$\vec{\upsilon}_x = \frac{d\vec{x}}{dt} \qquad \vec{\upsilon}_y = \frac{d\vec{y}}{dt} \qquad \vec{\upsilon}_z = \frac{d\vec{z}}{dt}$$

- **angular** $\omega = \dfrac{d\alpha}{dt}$

- **linear via angular** $\vec{\upsilon} = \dfrac{d\vec{r}}{dt} = \dfrac{d\alpha}{dt} \times \vec{r} = \omega \times \vec{r}$



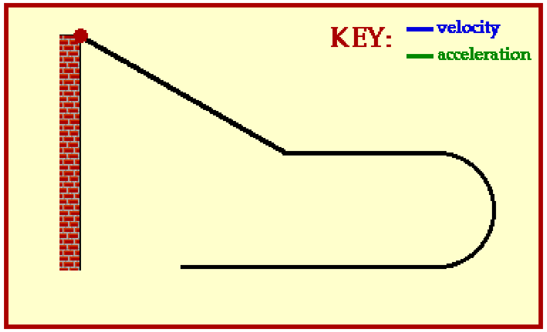
KINEMATICS OF MATERIAL POINT

ACCELERATION

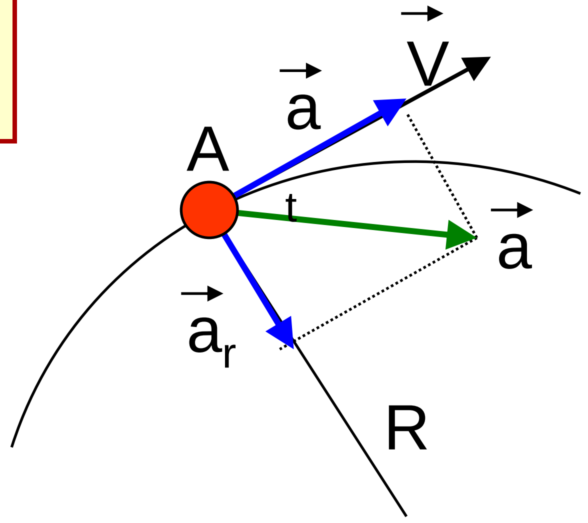
First derivative of vector velocity vs time – second derivative of vector position vs time

- **linear**

$$\vec{a} = \frac{d\vec{\upsilon}}{dt} = \frac{d^2\vec{r}}{dt^2}$$



- **radial and tangential**



- **angular** $\varepsilon = \frac{d\omega}{dt}$

- **linear via angular** $\varepsilon = \frac{a}{r}$

KINEMATICS OF MATERIAL POINT

GEOMETRICAL CLASSIFICATION OF MOTIONS

- **rectilinear - with constant:**

  - velocity $$x = x_o + \upsilon \cdot (t - t_o)$$

  - acceleration $$x = x_o + \upsilon_o \cdot (t - t_o) + \frac{1}{2} a \cdot (t - t_o)^2$$

    where: $$\upsilon - \upsilon_o = a \cdot (t - t_o)$$

- **angular - with constant:**

  - angular velocity $$\phi = \phi_o + \omega_o \cdot (t - t_o)$$

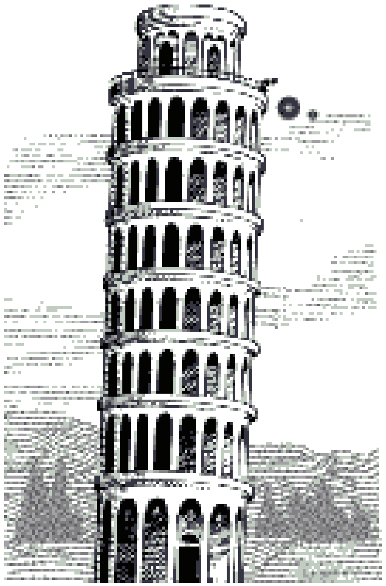
  - angular acceleration $$\phi = \phi_o + \omega_o \cdot (t - t_o) + \frac{1}{2} \varepsilon \cdot (t - t_o)^2$$

    where: $$\omega - \omega_o = \varepsilon \cdot (t - t_o)$$

KINEMATICS OF MATERIAL POINT

ACCELERATION DUE TO GRAVITY

- **FREE FALL**

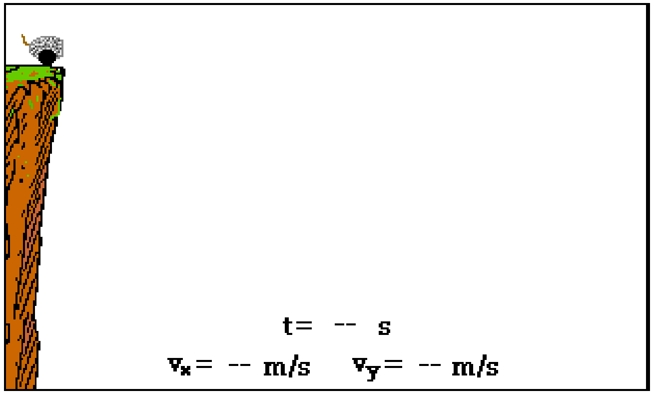


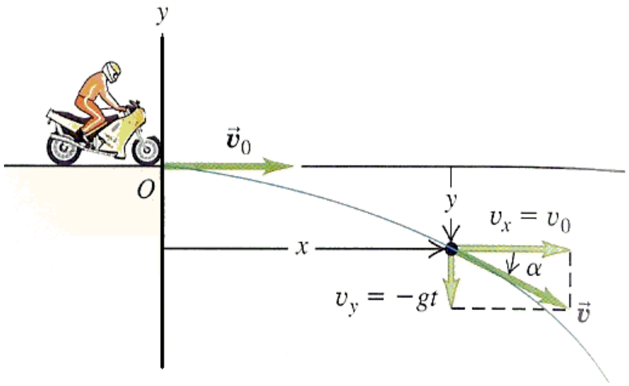
Basic relations of motion:

$$\upsilon_y = g \cdot t$$

$$y = \frac{1}{2} g \cdot t^2$$

- **HORIZONTAL THROW** (horizontal free fall) **Basic relations of motions:**





horizontal

$$x = \upsilon_o \cdot t$$

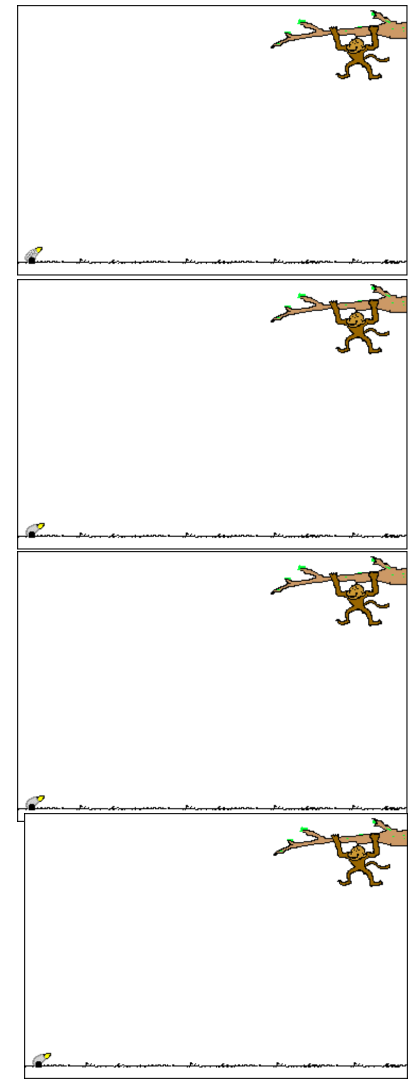
free fall

$$y = \frac{1}{2} g \cdot t^2$$

KINEMATICS OF MATERIAL POINT

ACCELERATION DUE TO GRAVITY

- **PROJECTILE THROW (motion at constant acceleration)**



Example:
Free falling monkey try to catch bananas:

Two boundary cases:

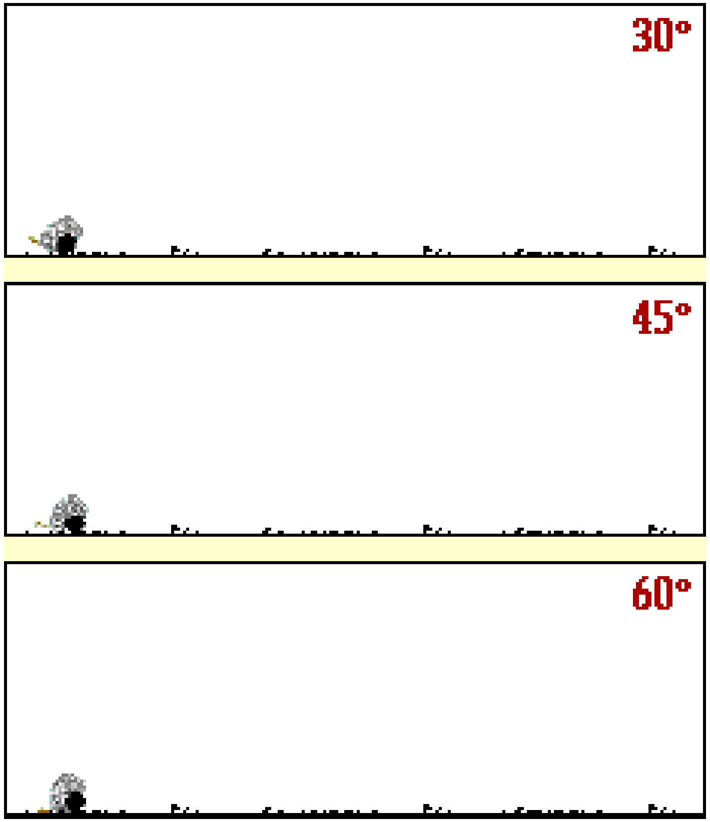
- **trajectories of motion of monkey and bananas never crosses!**

- **trajectories of motion of monkey and bananas crosses only at one proper time !**

KINEMATICS OF MATERIAL POINT

ACCELERATION DUE TO GRAVITY

- **PROJECTILE THROW (motion at constant acceleration)**



Basic relations

horizontal component

$$\upsilon_x = \upsilon_o \cdot \cos\alpha \qquad x = \upsilon_o \cdot t \cdot \cos\alpha$$

vertical component

$$\upsilon_y = \upsilon_o \cdot \sin\alpha - g \cdot t \qquad y = \upsilon_o \cdot t \cdot \sin\alpha - \frac{1}{2} g \cdot t^2$$

Boundary conditions:

Max. height at $\upsilon_y = 0 \rightarrow y_{max} = \dfrac{\upsilon_o^2 \cdot \sin^2\alpha}{2g}$

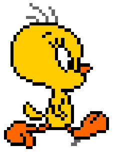
Max. horizontal distance at

$$y = 0 \rightarrow \quad x_m = \frac{\upsilon_o^2 \cdot \sin 2\alpha}{g}$$

DYNAMICS OF MASS IN SPACE

MOTION OF PARTICLE IN SPACE

- **forms – domain of kinematics**
- **causes – domain of dynamics**



Main simplification of kinematics: particle as material point $\vec{\upsilon}(t)$

Description of motion of particle in space: velocity vector in time
Any change in motion (position, velocity) of particle only possible after overcoming resistance of inertia

Measure of inertia: MASS of body (particle)
operational (physical) parameter: relative with respect to unit mass (1 kg)

Primary concept (parameter) of dynamics: MOMENTUM of a body

$$\vec{p} = m \cdot \vec{\upsilon} = m \frac{d\vec{r}}{dt}$$

Motion of a particle: localized transport of momentum (energy)

DYNAMICS OF MATERIAL POINT

CAUSE OF MOTION

Influence of other particle(s) or body (bodies) through a FORCE

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m \cdot \vec{\upsilon})}{dt} = \frac{dm \cdot \vec{\upsilon}}{dt} + \frac{m \cdot d\vec{\upsilon}}{dt}$$

what causes change of a particle position in space
In general, both two components should be taken into account –
domain of specific theory of relativity (Einstein – 1915)

However, commonly observed effects and related simplified assumption:
mass m = constant

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m \cdot \vec{\upsilon})}{dt} = m\frac{d\vec{\upsilon}}{dt} = m \cdot \vec{a}$$

Under unbalanced force acting a body (particle) attains an acceleration

$$\vec{a} = \frac{\vec{F}}{m}$$

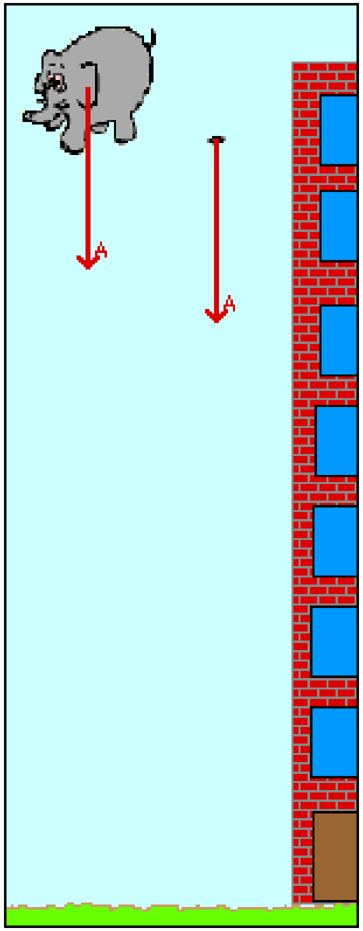
proportional to a force F, and inversely proportional to its mass m
- II Dynamics Principle – II Newton's law

DYNAMICS OF MATERIAL POINT

Basic question appears – is II Newton's law always justified ?

Answer: No (everyting depends on boundary assumption(s)

- **Free falling of elephant and feather (air resistance neglected)**



According to II Newton's law:

both bodies (of different shape and mass) attain an acceleration g proportional to weight G, and inversely proportional to its mass m

$$\vec{g} = \frac{\vec{G}}{m}$$

Final velocity only depends on time

$$\vec{\upsilon} = \vec{g} \cdot t$$

and is the same for both bodies !!!

However, a question appears: is it real (true) effect?

DYNAMICS OF MATERIAL POINT

- **Free falling of elephant and feather (air resistance occurs)**

According to II Newton's law:

Both bodies (of different shape and mass) attain the different forms of motion (acceleration)

Final motion (velocity, acceleration) strongly depends on air resistance

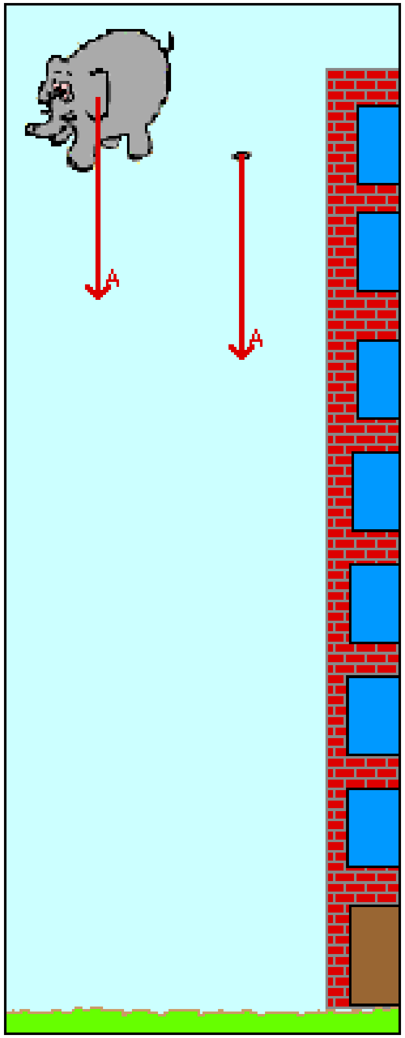
$$\vec{F}_{net} = G - F_{air}(\vec{\upsilon})$$

and is completely different for both bodies

$$\vec{\upsilon}_e \succ\succ \vec{\upsilon}_f$$

because air resistance only affects the feather's velocity, its final velocity

$$\vec{\upsilon}_f = const$$



DYNAMICS OF MATERIAL POINT

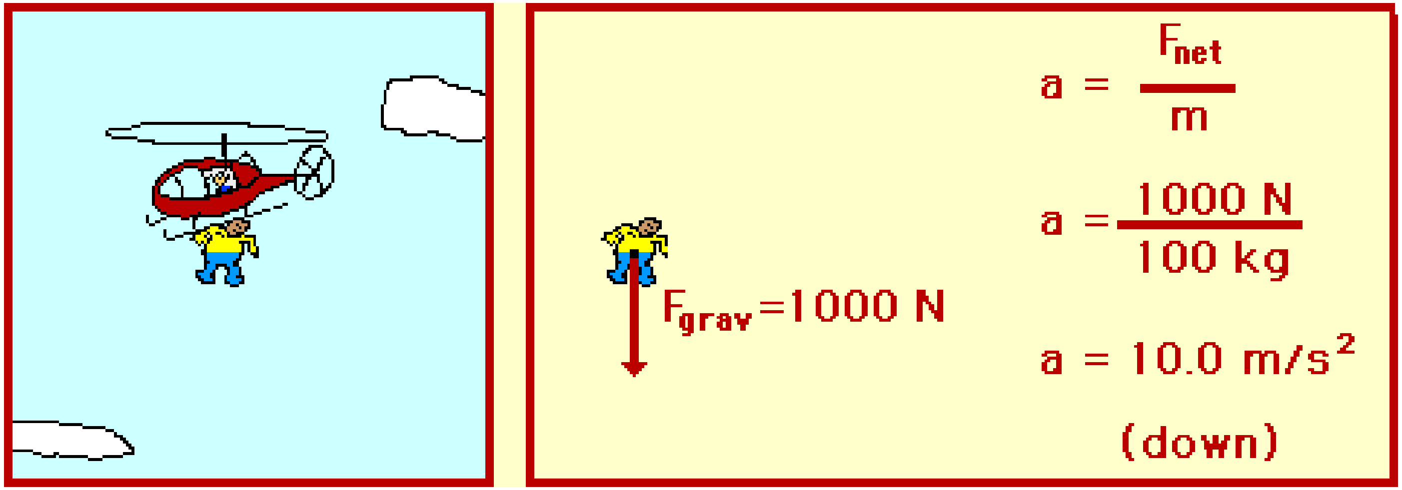
Important specific case:

when no force is acting on body, or all the forces acting are balanced - a resultant motion of particle at constant velocity vector in time

$$\vec{\upsilon}(t) = const$$

It concerns s.c. isolated particle - I Dynamics Principle – I Newton's law

- **free falling of man (woman) from plane on parachute**



Two step motion:

- at constant a = g - free fall under gravitation $\vec{F}_g = m \cdot \vec{g}$ **and** $\vec{\upsilon}_g = \vec{g} \cdot t$

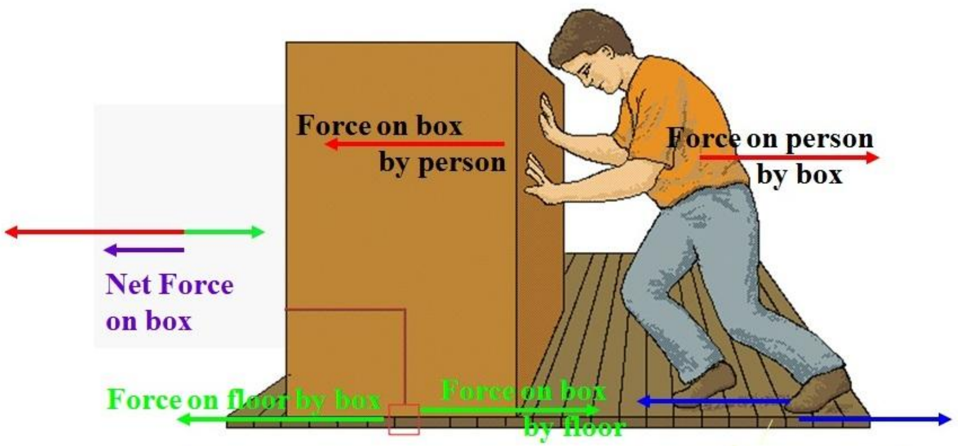
- at constant υ - free fall compensated by air resistance $\vec{F}_{air} = 6\pi \cdot \eta \cdot r \cdot \vec{\upsilon}$

DYNAMICS OF MATERIAL POINT

Motion of body affected not only by surrounding (medium) – influnce of other (external) bodies – common effect

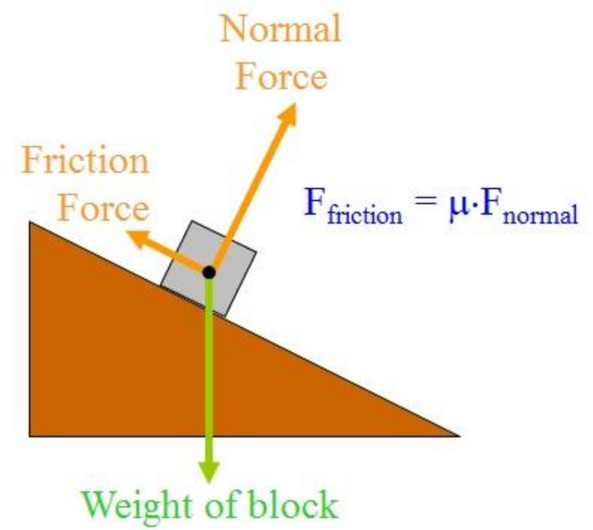
- **motion of body in presence of friction**

plane (perpendicular to weight)



A pair of forces acting between person and floor.

declined plane



Description of motion:

relations of motion (II Newton's law) regarding all the forces acting along a specific direction of motion

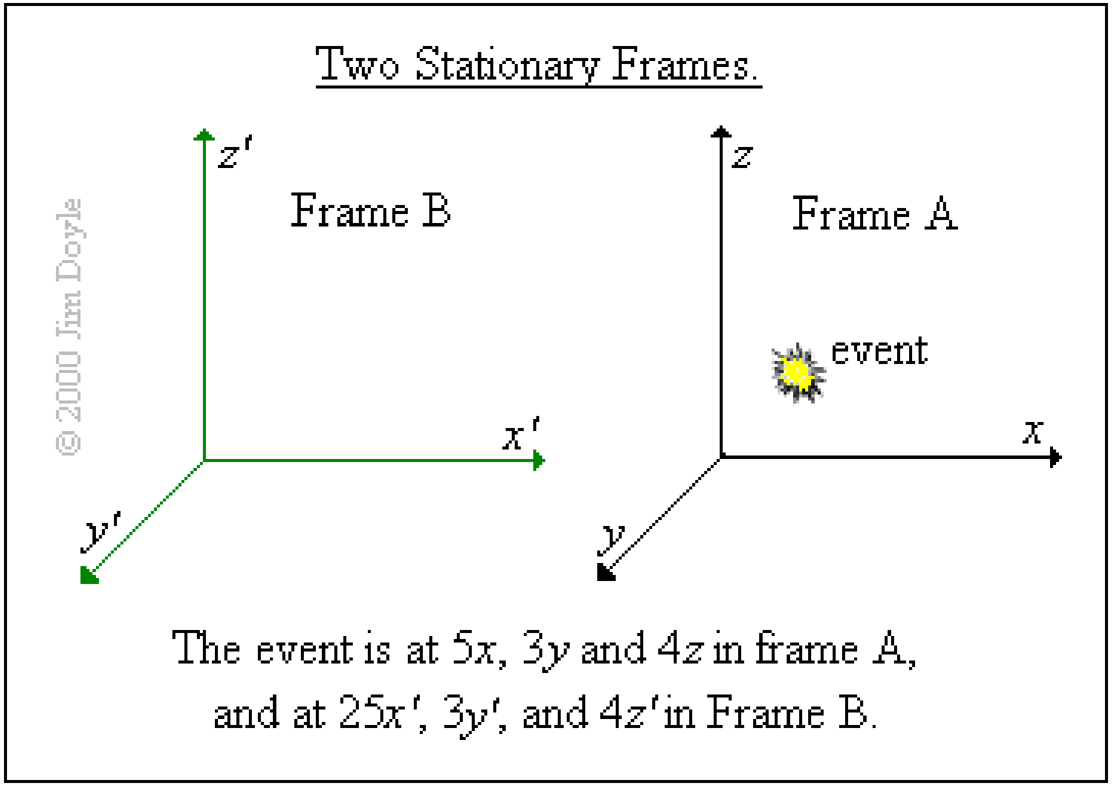
$$\vec{F}_{net} = \sum \vec{F}_{comp}$$

DYNAMICS OF MATERIAL POINT

Motion of body affected not only by surrounding (medium) – influnce of reference systems (frames) – common effects:

- **two stationary frames**

Event occurs identically in both frames at different coordinates, i.e. motion of particle is invariant in both frames - Galileo transformation

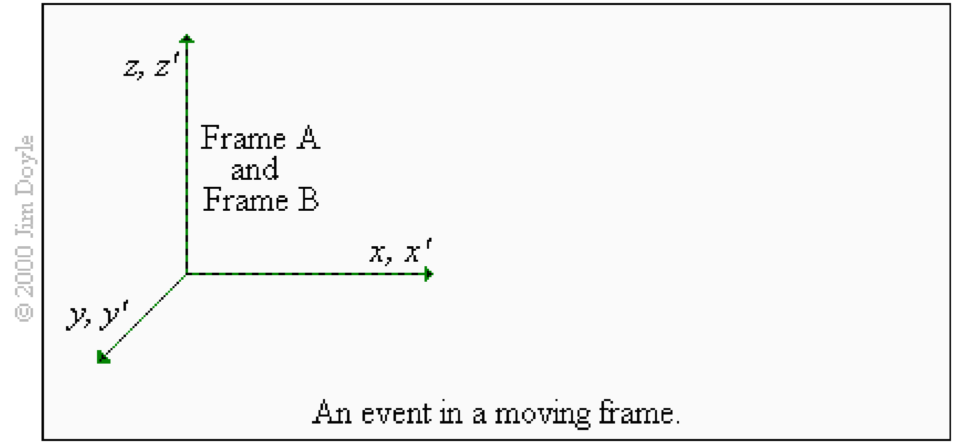


There are two inertial systems (frame(s)) – during transition between them dynamics are invariant – I Newton's law is satisfied

DYNAMICS OF MATERIAL POINT

- **relative motion of frame(s) at constant drift velocity**
  during a transition between them - transformation of:

- **position:** $x' = x - \upsilon \cdot t$
- **velocity:** $\upsilon' = \upsilon - \upsilon_{drift}$
- **acceleration:** $a' = a$



An event in a moving frame.

Event occurs as invariant in both frames - I Newton's law is also satisfied

Examples:
- **motion of particle in vagon moving at constant velocity**
- **relative motion of two paralel vagons at railway station (ship on a sea)**

However, when I Newton's law is not satisfied - non-inertial frame - motion of particle at constant acceleration under additional inertial force

There are two different general cases – inertial forces:
- **at translatory motion**
- **at rotation of reference system**

DYNAMICS OF MATERIAL POINT

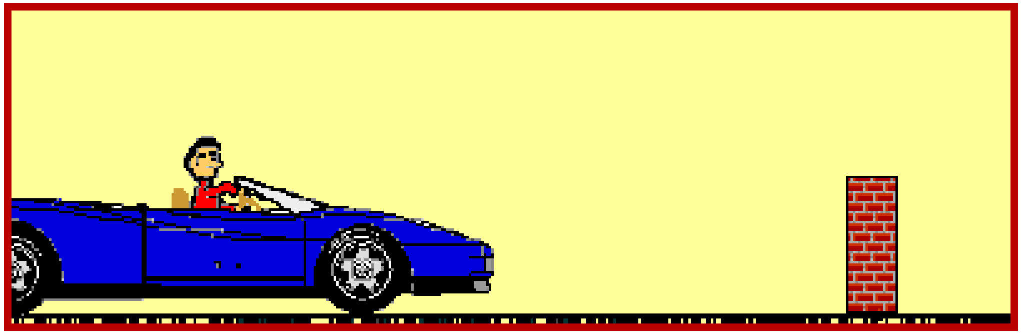
INERTIAL FORCES AT TRANSLATORY MOTION

Inertial frame – motionless body or at constant $\vec{\upsilon}$ **thus** $\vec{a}_i = 0$

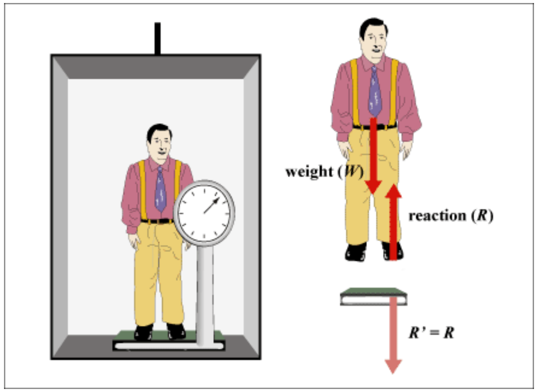
Non-inertial frame - uniformly accelerated motion at constant $\vec{a}_o$

Examples:

- **Car collision with a wall**



Elevator in motion



Bodies in both systems move with an acceleration $\vec{a}_i = -\vec{a}_o$
satisfying a condition

$$\vec{a}_i = \vec{a}_o + \vec{a}_n$$

Resultant effect: additional force is acting - inertial force

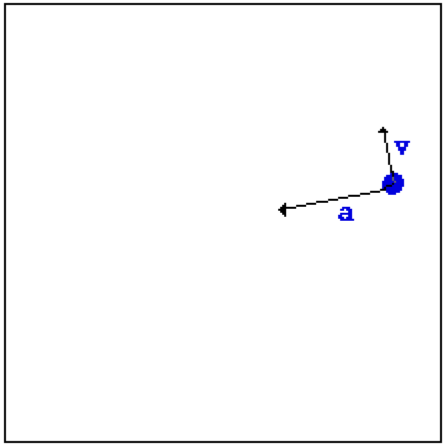
Rule:

Inertial force always turned against the acceleration of particle (system) – attempt to conserve a previous form of motion (constant velocity) $\vec{\upsilon}$

DYNAMICS OF MATERIAL POINT

NON-INERTIAL FORCE AT ROTATION

Non-inertial primary force at rotation - Centripetal force
Motion of particle of mass m in a circle of radius r toward central point (centre seeking force)

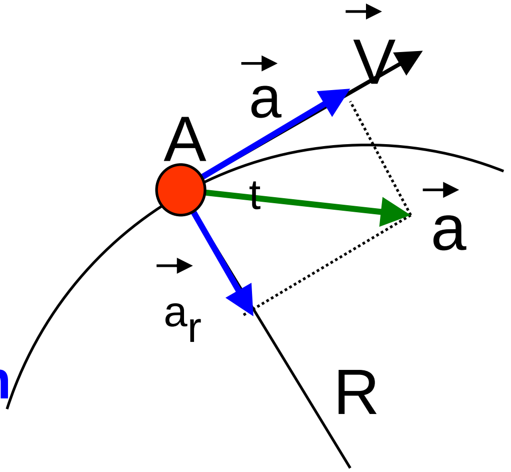


Two components:

- **radial (normal):** $$F_r = m \cdot \vec{a}_r = m \cdot \vec{\omega} \times \vec{\upsilon}$$
- **tangential** $$F_t = m \cdot \vec{a}_t = m \cdot \vec{\varepsilon} \times \vec{r}$$

Total acceleration of motion in circle

$$a_{rot} = \vec{a}_r + \vec{a}_t$$



Examples:

- **gravitational attraction: Earth - Moon**

$$F_{grav} = G \frac{M_E \cdot m_M}{r_{EM}^2}$$

- **electrostatic attraction: electron - nuclei in H atom**

$$F_C = \frac{1}{4\pi\varepsilon_o} \frac{Q_n \cdot q_e}{r_{ne}^2}$$

DYNAMICS OF MATERIAL POINT

TRUE INERTIAL FORCES AT ROTATIONAL MOTION

• Centrifugal force

Motion of particle of mass m inside (within) reference frame rotating at constant angular velocity ω along radius r outward a central point

$$F_{cf} = -m \cdot \omega \times (\omega \times r)$$

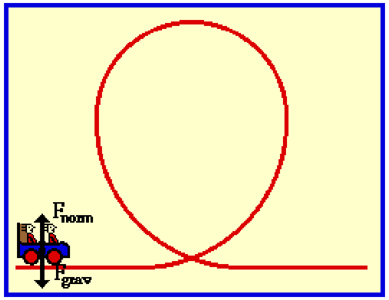
Only for simplified case: radius ⊥ to axis of rotation

$$F_{cf} = -m \cdot \omega^2 \cdot r$$

Examples:

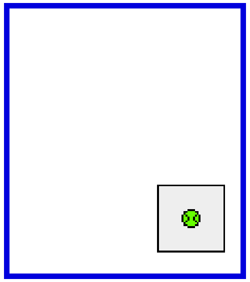
- **Stagnant loop**

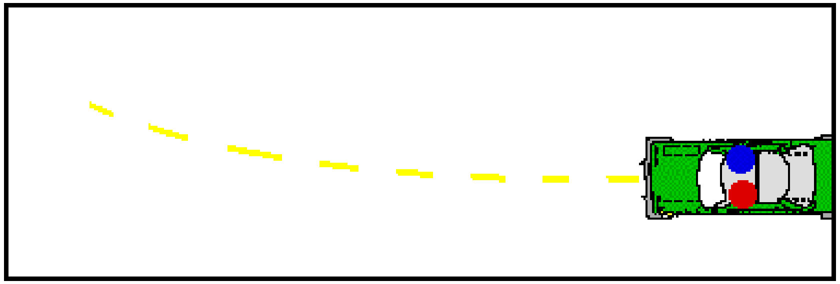
at highest point balance between cetrifugal force and weight of a body



- **Car at the corner (turning)**

motion still along straight line - deflection with respect to the car primary trajectory





DYNAMICS OF MATERIAL POINT

TRUE INERTIAL FORCES AT ROTATIONAL MOTION

• Coriolis force

Motion of particle of mass m at velocity υ with respect (outside) to reference frame rotating at constant angular velocity ω

Resultant effect - moving of a body along distance

$$s = d \cdot \varphi = \upsilon \cdot t \cdot \omega \cdot t \cdot = \upsilon \cdot \omega \cdot t^2 = \frac{1}{2} a_C \cdot t^2$$

with acceleration

$$a_C = \frac{2s}{t^2} = 2\upsilon \cdot \omega$$

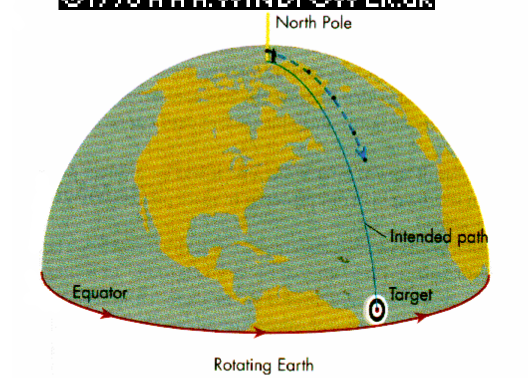
under Coriolis force

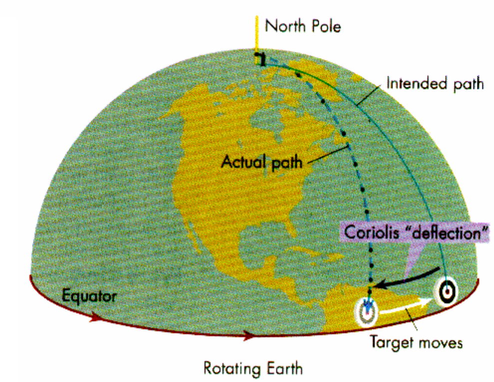
$$\vec{F}_{Cor} = m \cdot \vec{a}_{Cor} = 2m(\vec{\upsilon} \times \vec{\omega})$$

Resultant effect: deflection of particle trajectory

Primary condition: $\upsilon > 0$;

Max. value at $\upsilon \perp \omega$ (equator)



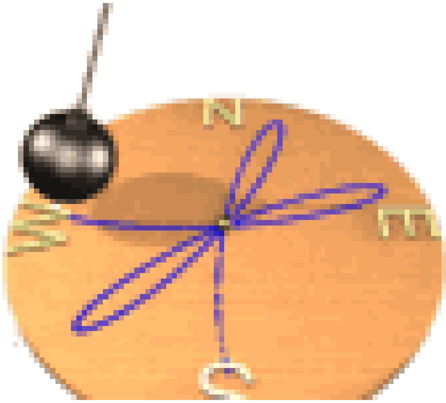


DYNAMICS OF MATERIAL POINT

TRUE INERTIAL FORCES AT ROTATIONAL MOTION

• Coriolis force

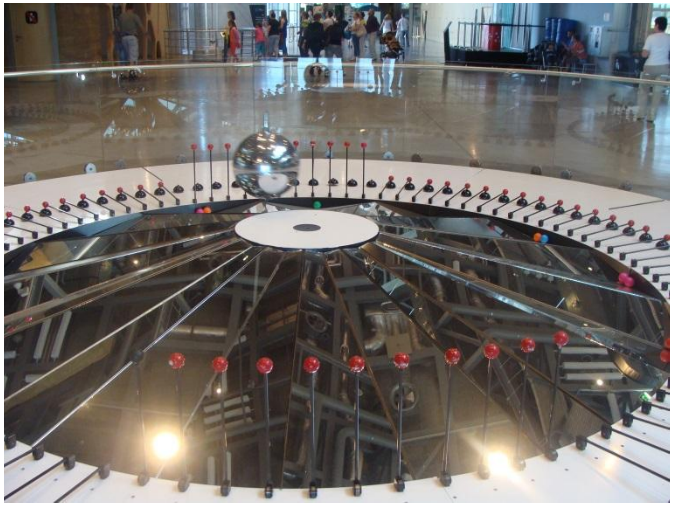
Experimental demonstration
Foucault pendulum (NDC Paris - 1850)

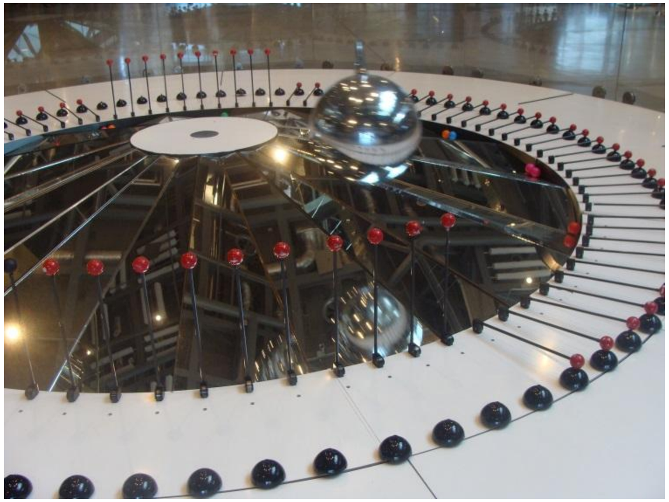


Hesitation of simple pendulum with respect to the rotating target corresponding to rotating Earth
Resultant rotation of hesitation plane - loop

Gdansk Univ.Techn.

Copernicus Centre Warsaw - entrance





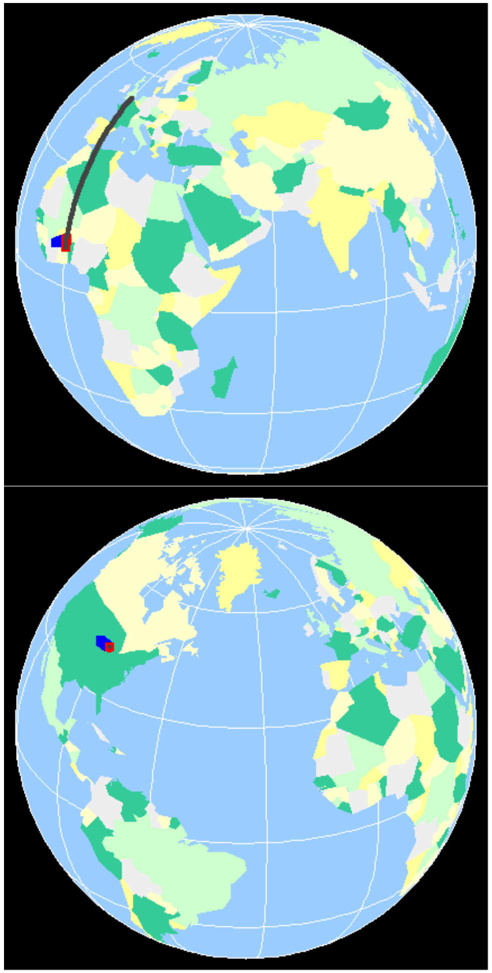
DYNAMICS OF MATERIAL POINT

TRUE INERTIAL FORCES AT ROTATIONAL MOTION

• Coriolis force

Common effect: motion of a body with respect to rotating Earth

Free falling of body near Earth



Deflection of trajectory of a body free falling in plumb-line to Earth - direction depends on hemisphere:

- **northern: to the East**
- **southern: to the West**

For free falling from h ~ 1 km - deflection s ~ 30 cm
Example:
vertical departure of a rocket (satelite) –
TV observation from Cape Canaveral (Florida, USA)

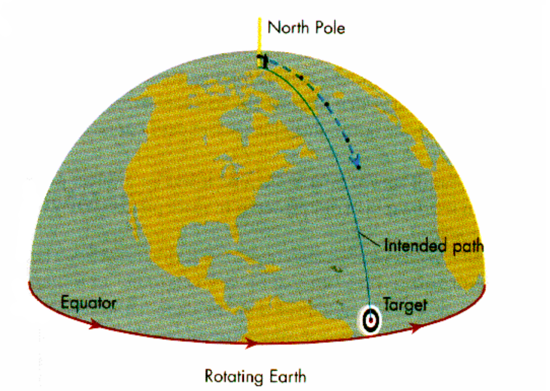
DYNAMICS OF MATERIAL POINT

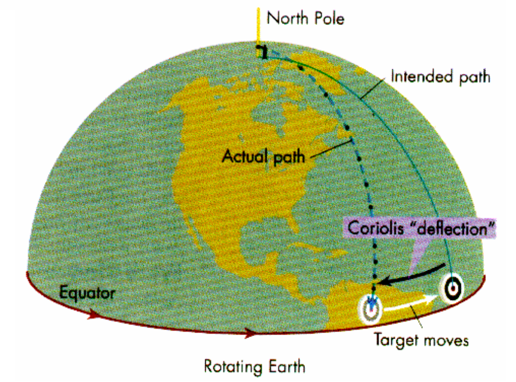
TRUE INERTIAL FORCES AT ROTATIONAL MOTION

• Coriolis force

Common effect: motion of a body with respect to rotating Earth

Interaction of water flow with a river bank





dependent on hemisphere and geographical direction of water flow:

- **northern hemisphere and southern flow: right bank**
- **northern hemisphere and northern flow: left bank**

Outflow of water from sink (bathtub)

dependent on hemisphere - water flow down:

On our northern hemisphere: resultant left whirl

DYNAMICS OF MATERIAL POINT

TRUE INERTIAL FORCES AT ROTATIONAL MOTION

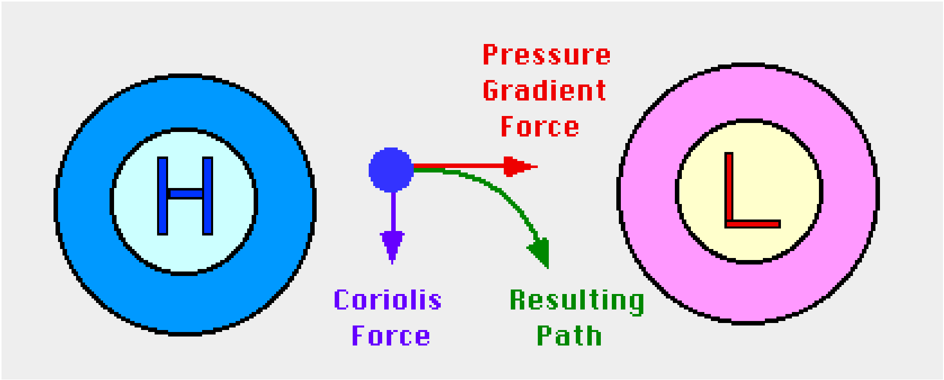
• Coriolis force

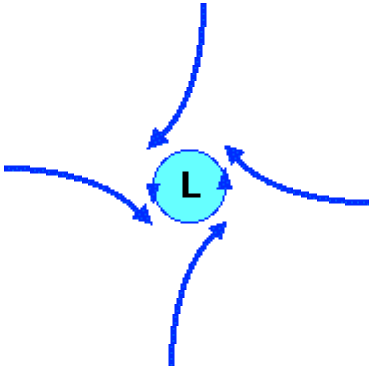
Common effect: motion of a body with respect to rotating Earth

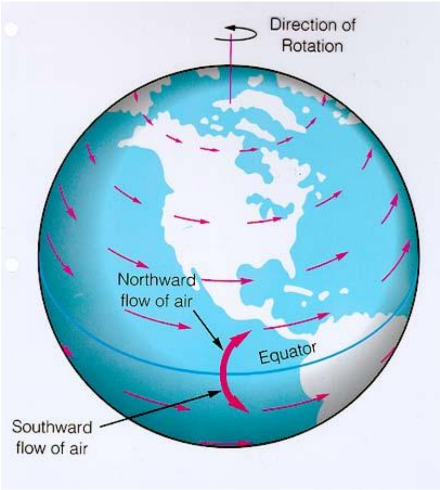
Meteorological phenomenon

dependent on hemisphere and local position of high and low pressure centers, respectively

Northern hemisphere







Resultant direction of air flux (wind) depends on type of pressure centre

Base for weather forecast (!)