CONSERVATION PRINCIPLES OF MATERIAL POINT

Main simplification of dynamics - particle as material point

Motion of particle in space determined by velocity vector in time

Primary concept of dynamics: MOMENTUM of body

$$\vec{p} = m \cdot \vec{\upsilon} = m \frac{d\vec{r}}{dt}$$

For translatory motion of particle:
change of particle position under influence of other particle(s) or system via force(s)

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m \cdot \vec{\upsilon})}{dt} = m \frac{d\vec{\upsilon}}{dt} = m \cdot \vec{a}$$

Motion of particle - localized transport of:

- **momentum**
- **angular momentum**
- **energy**

determined by the respective conservation principles.

CONSERVATION PRINCIPLE OF MOMENTUM

Motion of particle determined by velocity vector in time

When no force is acting on body, or all the forces acting are balanced thus

$$\vec{\upsilon}(t) = const$$

- case of isolated particle: First Dynamics Principle - NEWTON'S I LAW

If no net external force acts on a particle

$$\sum_{i=1}^{n} F_i^{ext} + \sum_{i=1}^{n} F_i^{int} = 0$$

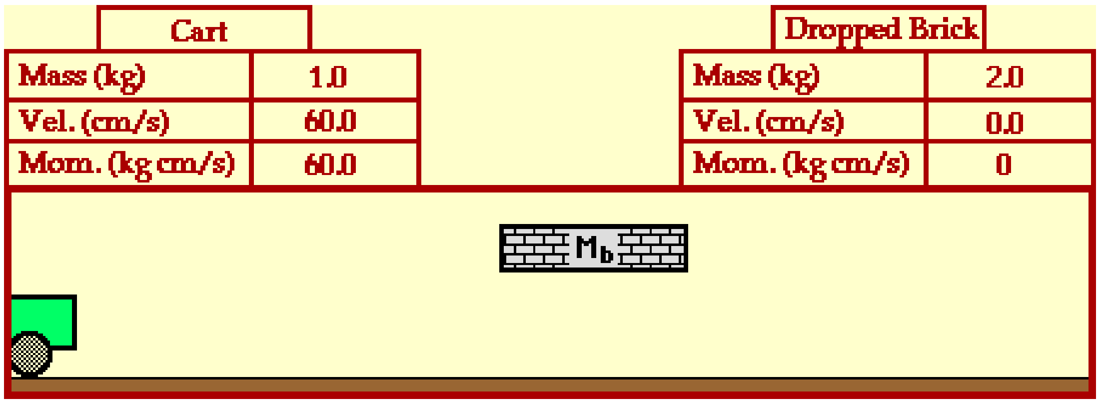
a total momentum of particle remains constant - principle of conservation of momentum

$$\vec{p}_t = \sum_{i=1}^{n} p_i = const$$

CONSERVATION PRINCIPLE OF MOMENTUM

EXAMPLE 1:

- **Isolated system: cart and dropped brick**



- before brick dropping:

$$M_C \cdot \upsilon_C \neq 0$$

- after brick dropping:

$$M_C \cdot \upsilon_C = (M_C + M_B) \cdot \upsilon_r$$

Resultant velocity of system:

$$\upsilon_r = \frac{M_C \cdot \upsilon_C}{(M_C + M_B)}$$

CONSERVATION PRINCIPLE OF MOMENTUM

EXAMPLE 2:

• **Isolated system: man on a boat**

- boat and man in rest - steady state

$$p_t = (m_B + m_M) \cdot \upsilon = 0$$

- man jumping (escaping) from a boat

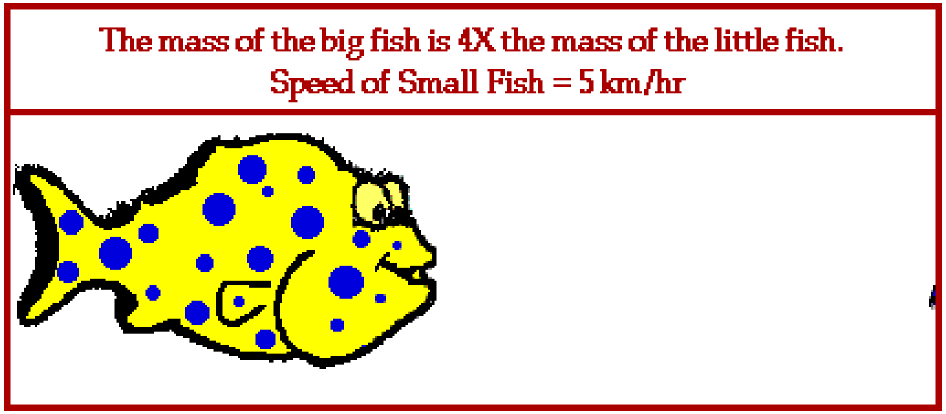
$$p_t = m_B \cdot \upsilon_B - m_M \cdot \upsilon_M = 0$$

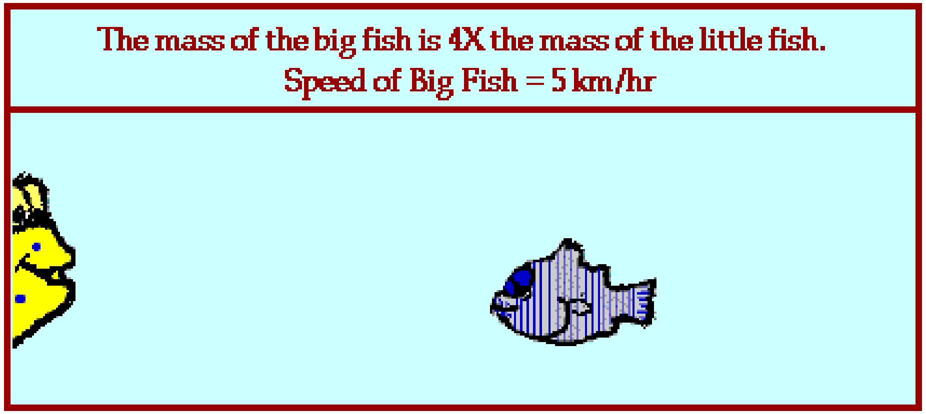
Resultant effect: motion of boat and man in opposite direction -
Third Dynamics Principle – Newton's III Law

CONSERVATION PRINCIPLE OF MOMENTUM

EXAMPLE 3:

• **Fully inelastic „collision" of two fishes**





Before swallow:

Big fish: $m_{BF}\cdot\upsilon_{BF}=0$ **Small fish:** $m_{SF}\cdot\upsilon_{SF}\neq 0$

After swallow:

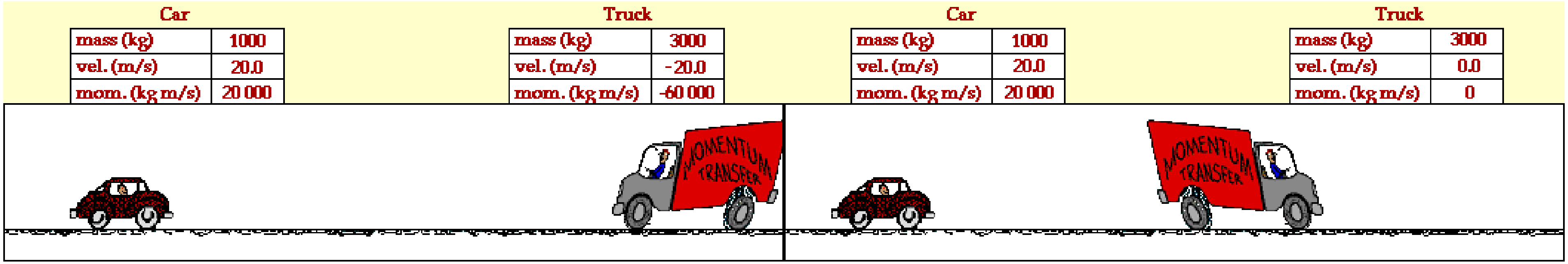
Joined system (small fish inside big fish):

$$m_{BF}\cdot\upsilon_{BF}(0)+m_{SF}\cdot\upsilon_{SF}=(m_{BF}+m_{SF})\cdot\upsilon_{r}$$

CONSERVATION PRINCIPLE OF MOMENTUM

EXAMPLE 4:

- **Fully inelastic collisions: car and track**



Car	
mass (kg)	1000
vel. (m/s)	20.0
mom. (kg m/s)	20 000

Truck	
mass (kg)	3000
vel. (m/s)	-20.0
mom. (kg m/s)	-60 000

Car	
mass (kg)	1000
vel. (m/s)	20.0
mom. (kg m/s)	20 000

Truck	
mass (kg)	3000
vel. (m/s)	0.0
mom. (kg m/s)	0

Before collision:

Car: $m_C \cdot \upsilon_C \neq 0$ **Track:** $m_T \cdot \upsilon_T(-) \neq 0$

After collision:

Joined system:

$$m_C \cdot \upsilon_C - m_T \cdot \upsilon_T = -(m_C + m_T) \cdot \upsilon_r$$

CONSERVATION PRINCIPLE OF MOMENTUM

EXAMPLE 5:

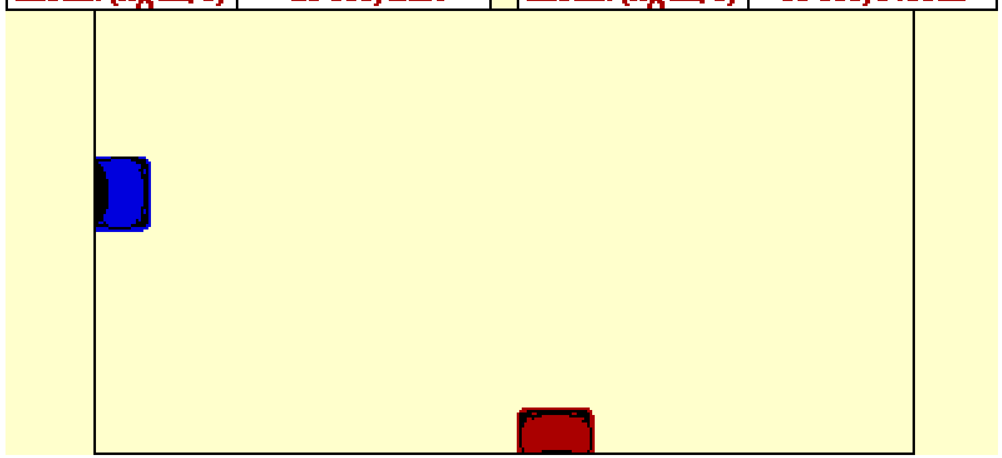
• **Fully inelastic collisions of two cars at side crash**

Blue Car

mass (kg)	1000
vel. (m/s)	20.0, East
mom. (kg m/s)	20 000, East

Red Car

mass (kg)	1000
vel. (m/s)	10.0, North
mom. (kg m/s)	10 000, North



Before collision:

Blue car: $m_{BC}\cdot\upsilon_{BC}\neq 0$ **Red car:** $m_{RC}\cdot\upsilon_{RC}\neq 0$

After collision:

joined system

$$m_{BC}\cdot\upsilon_{BC}+m_{BC}\cdot\upsilon_{RC}=(m_{RC}+m_{BC})\cdot\upsilon_r$$

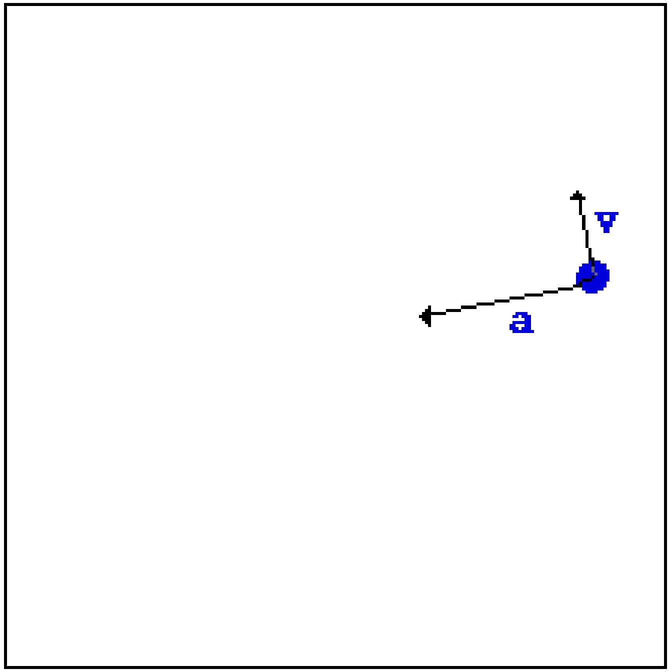
CONSERVATION PRINCIPLE OF ANGULAR MOMENTUM

ANGULAR MOMENTUM

Appear at rotary motion of particle

$$\vec{L} = m \cdot \vec{r} \times \vec{\upsilon} = \vec{r} \times \vec{p}$$

Under influence of torque (moment of force) - force applied on particle with respect to axis of rotation of reference system – change of angular momentum of particle



$$\vec{M} = \vec{r} \times \vec{F} = \vec{r} \times \frac{d\vec{p}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p}) = \frac{d\vec{L}}{dt}$$

When no torque is acting on body angular momentum of particle remains constant

$$\vec{L} = \vec{r} \times \vec{p} = const$$

Principle of conservation of angular momentum

CONSERVATION PRINCIPLE OF ANGULAR MOMENTUM

EXAMPLE 1:

- **Planar motion of planet in Solar System**

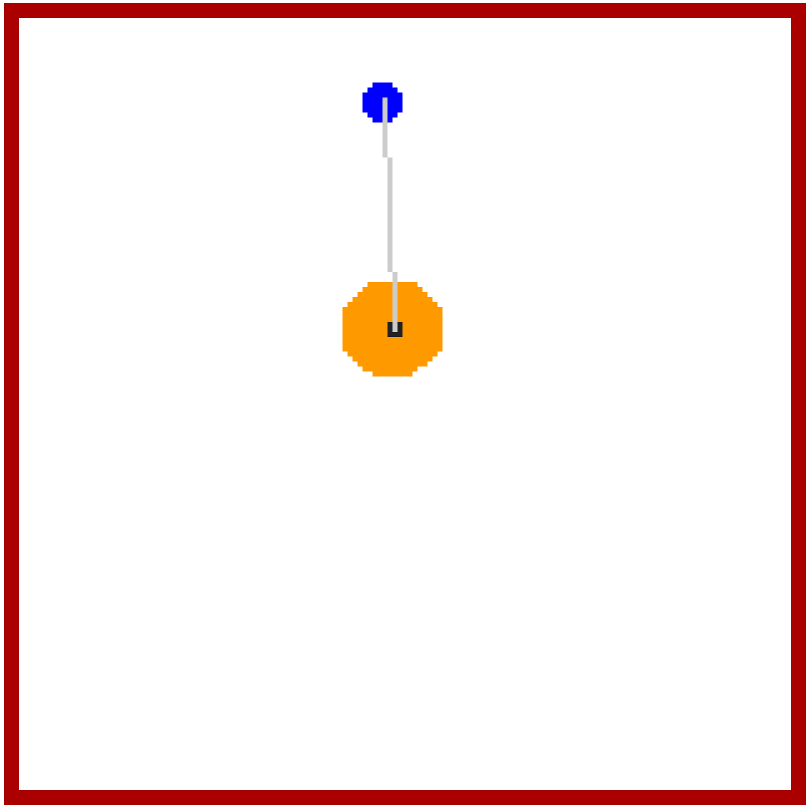
Constancy
of angular momentum of particle

$$\vec{L} = \vec{r} \times \vec{p} = const$$

Costancy
of sweep velocity during motion on orbit

$$\vec{r} \times \vec{\upsilon} = const$$

II Kepler's law



CONSERVATION PRINCIPLE OF MECHANICAL ENERGY

Forms of mechanical energy of particle:

- Kinetic energy

Energy of particle in motion - mechanical work done by force *F(s)* at distance *ds*

$$W = \int_o^s F \cdot ds = \int_o^s m \frac{d\upsilon}{dt} \cdot ds = m \int_o^\upsilon \frac{ds}{dt} d\upsilon = m \int_o^\upsilon \upsilon \cdot d\upsilon = \frac{1}{2} m \cdot \upsilon^2 = E_k$$

For isolated system: total kinetic energy constant

- Potential energy

Energy of particle determined by its position in space (field) – mechanical work required to change a position

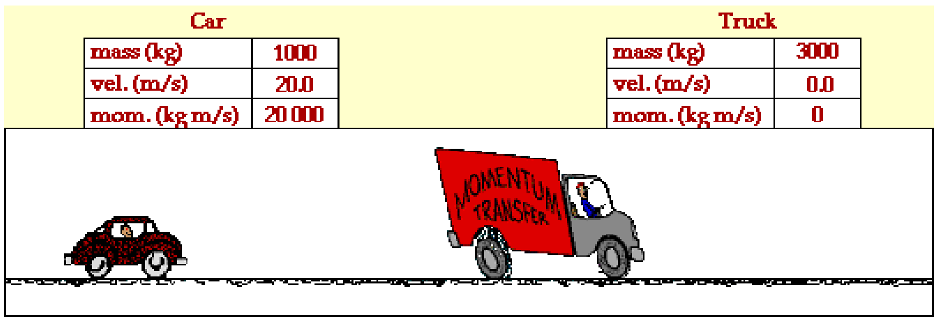
For gravitation:

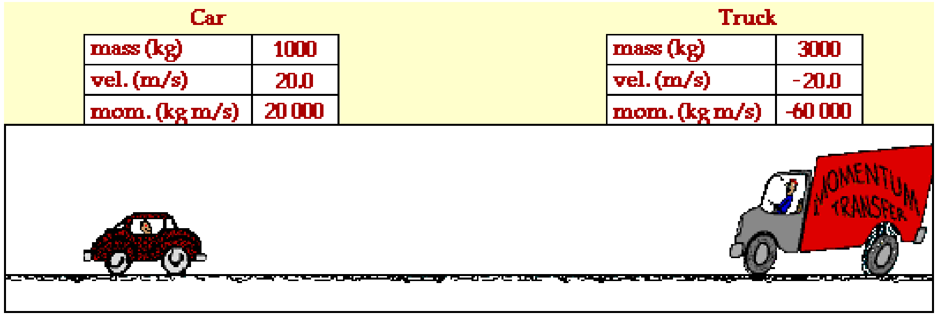
$$W = \int_o^s F \cdot ds = \int_o^s m \cdot g \cdot ds = m \cdot g \int_o^s ds = m \cdot g \int_o^h dh = m \cdot g \cdot h = E_p$$

CONSERVATION OF MOMENTUM AND ENERGY

EXAMPLE 1:

- **Fully elastic collisions: car and track**





Before collision:

Car: $m_C \cdot \upsilon_C \neq 0$ **Track:** $m_T \cdot \upsilon_T \neq 0$

After collision:

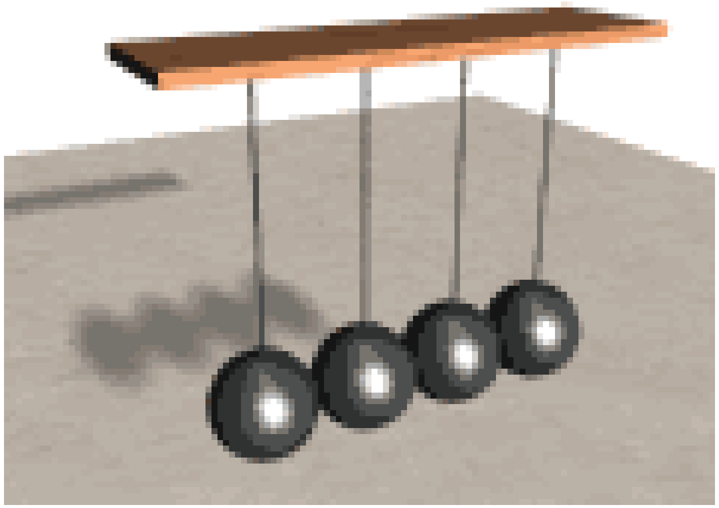
$$m_C \cdot \upsilon_{C1} + m_T \cdot \upsilon_{T1} = -m_C \cdot \upsilon_{C2} + m_T \cdot \upsilon_{T2}$$

Kinetic energy after collision:

$$\frac{1}{2} m_C \cdot \upsilon_{C1}^2 + \frac{1}{2} m_T \cdot \upsilon_{T1}^2 = \frac{1}{2} m_C \cdot \upsilon_{C2}^2 + \frac{1}{2} m_T \cdot \upsilon_{T2}^2$$

CONSERVATION OF MOMENTUM AND ENERGY

EXAMPLE 2:



- **Balls on threads (Newton's pendulum)**
  (Fully elastic collisions)

At collision:

momentum of right ball ↔ momentum of left ball

$$m_{RB} \cdot \upsilon_{RB} = m_{LB} \cdot \upsilon_{LB}$$

kinetic energy of right ball ↔ kinetic energy of left ball

$$\frac{1}{2} m_{RB} \cdot \upsilon_{RB}^2 = \frac{1}{2} m_{LB} \cdot \upsilon_{LB}^2$$

Principle of conservation of mechanical energy

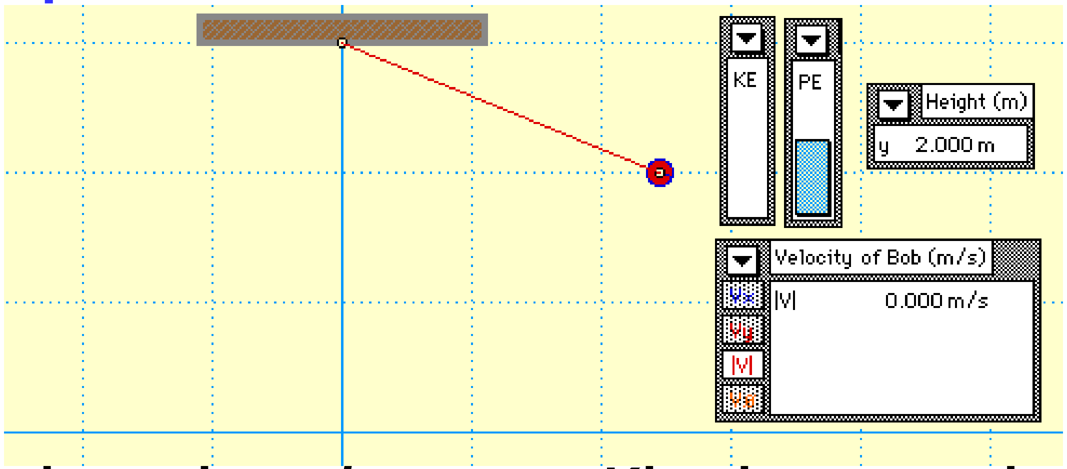
CONSERVATION PRINCIPLE OF MECHANICAL ENERGY

For isolated system: total mechanical energy is constant

$$E_k + E_p = const$$

EXAMPLE 1:

- **Mathematical pendulum**



Potential energy depends on *h*

$$E_p = m \cdot g \cdot h$$

Kinetic energy depends on

$$\upsilon = \sqrt{2g \cdot h}$$

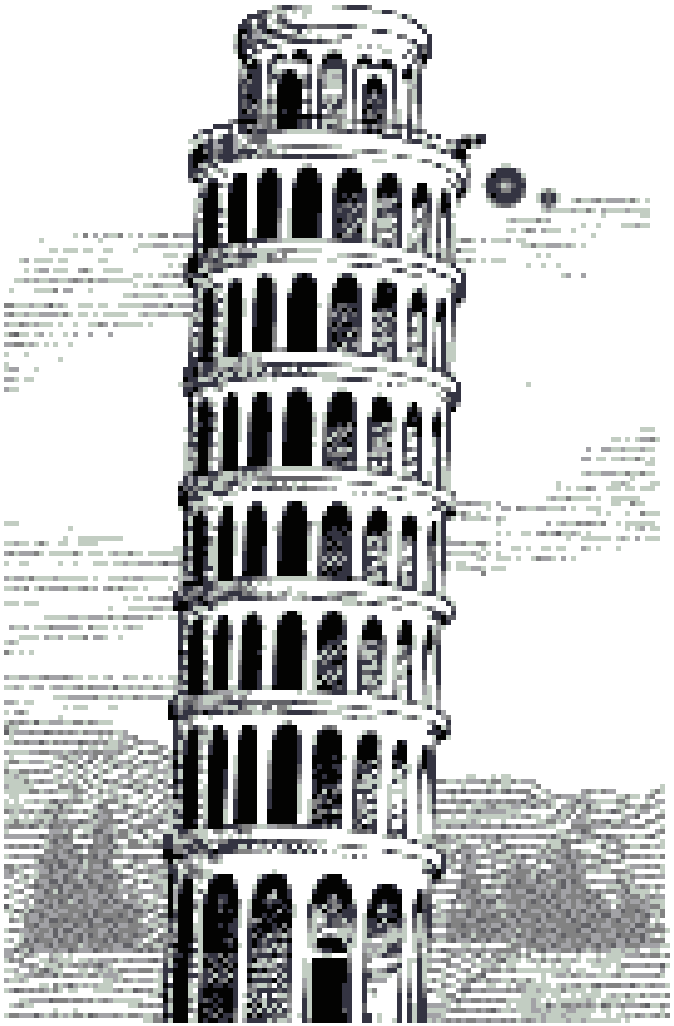
During a motion: exchange of forms of energy according to the condition

$$E_p + E_k = m \cdot g \cdot h + \frac{1}{2} m \cdot \upsilon^2 = const$$

CONSERVATION PRINCIPLE OF MECHANICAL ENERGY

EXAMPLE 2:

- **Free falling of ball from bent Pisa tower (air resistance neglected)**



At top: potential energy (max)

$$E_p = m \cdot g \cdot h$$

At bottom: kinetic energy (max)

$$E_k = \frac{1}{2} m \cdot \upsilon^2$$

During a motion: exchange of forms of energy according to the condition

$$E_p + E_k = m \cdot g \cdot h + \frac{1}{2} m \cdot \upsilon^2 = const$$

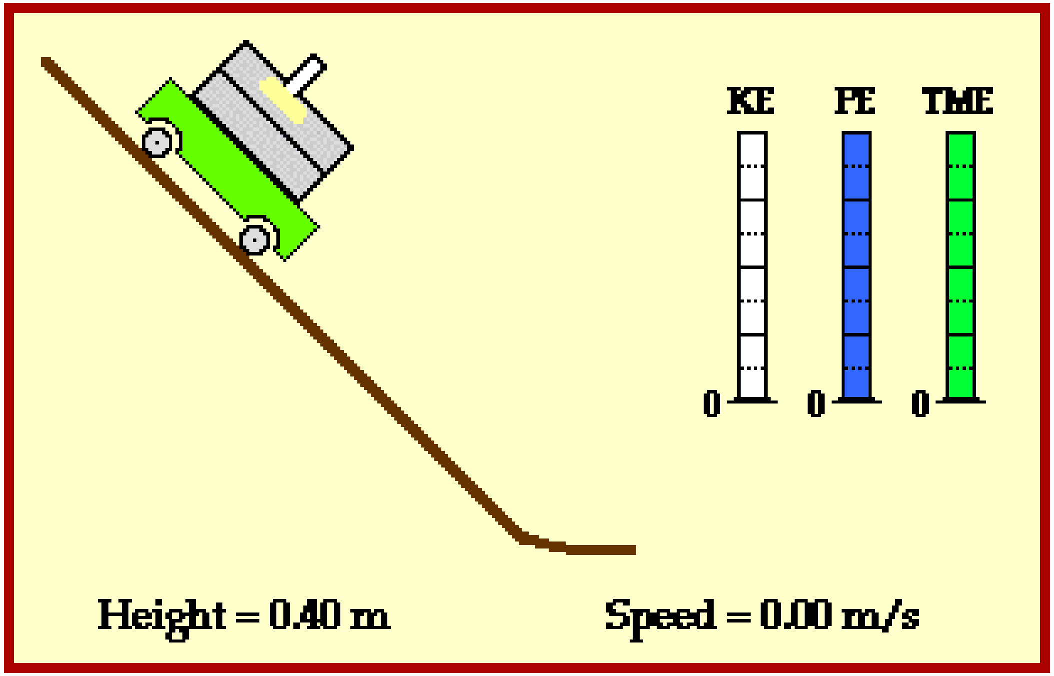
Resultant velocity $\upsilon = \sqrt{2g \cdot h}$

Typical behaviour for field of conservative forces

CONSERVATION PRINCIPLE OF MECHANICAL ENERGY

EXAMPLE 3:

- **Car descent from slope**



At top: potential energy (max)

$$E_p = m \cdot g \cdot h$$

At bottom: kinetic energy (max)

$$E_k = \frac{1}{2} m \cdot \upsilon^2$$

During a motion: exchange of forms of energy according to the condition

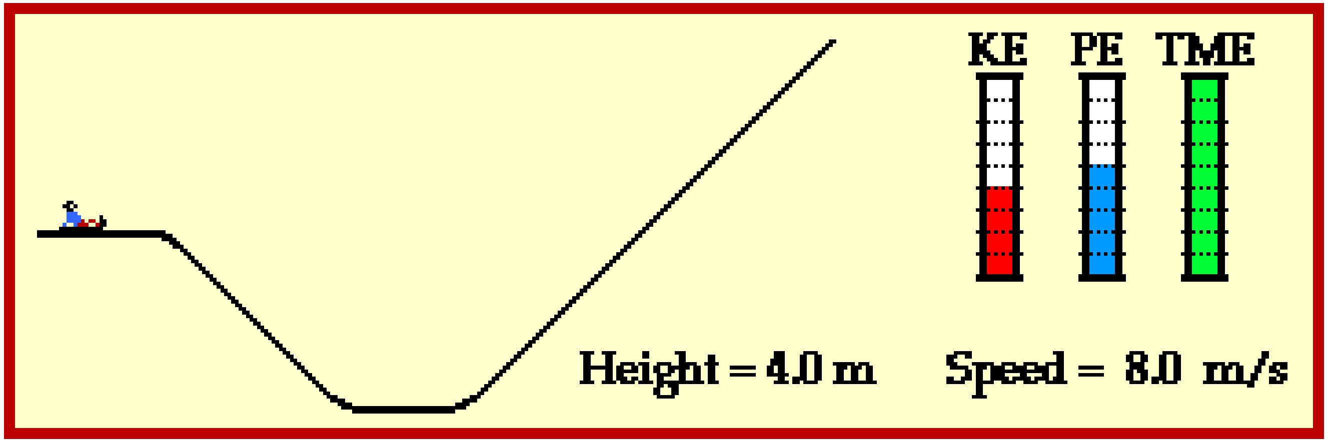
$$E_p + E_k = m \cdot g \cdot h + \frac{1}{2} m \cdot \upsilon^2 = const$$

Typical behaviour for field of conservative forces

CONSERVATION PRINCIPLE OF MECHANICAL ENERGY

EXAMPLE 4:

- Car motion on slope



Starting kinetic energy at the top $$E_{k1} = \frac{1}{2} m \cdot \upsilon_1^2$$

Starting potential energy at the top $$E_{p1} = m \cdot g \cdot h_1$$

During a motion: exchange of forms of energy according to the condition

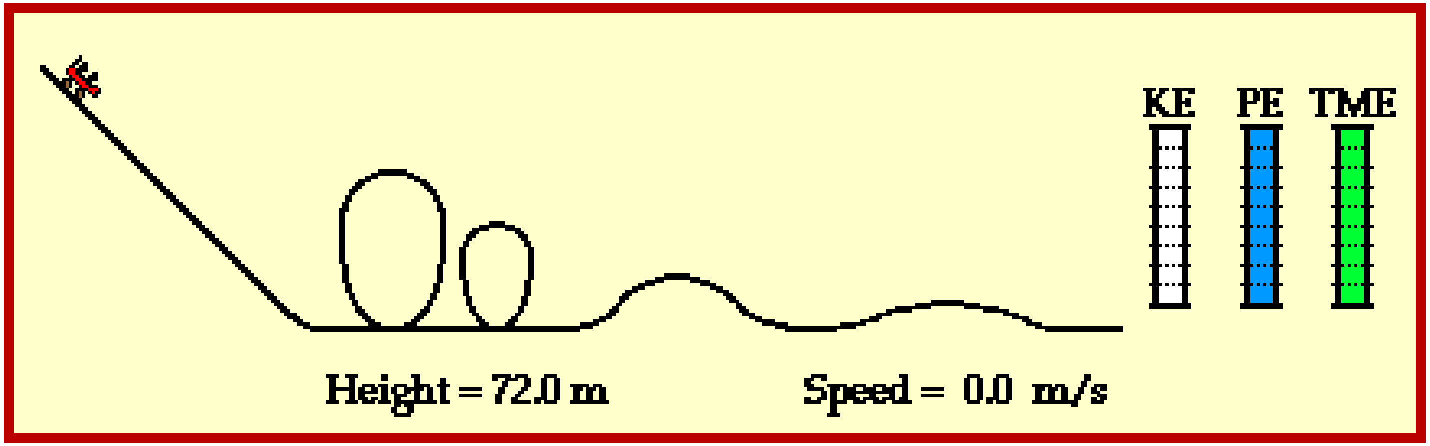
$$m \cdot g \cdot h_1 + \frac{1}{2} m \cdot \upsilon_1^2 = m \cdot g \cdot h_2 + \frac{1}{2} m \cdot \upsilon_2^2 = const$$

Typical behaviour for field of conservative forces

CONSERVATION PRINCIPLE OF MECHANICAL ENERGY

EXAMPLE 5:

- **Car motion on sequential loops**



Potential energy depends on *h*

$$E_p = m \cdot g \cdot h$$

Kinetic energy depends on

$$\upsilon = \sqrt{2g \cdot h}$$

During a motion: exchange of forms of energy according to

$$E_p + E_k = m \cdot g \cdot h + \frac{1}{2} m \cdot \upsilon^2 = const$$

Typical behaviour for field of conservative forces