WAVES PROPAGATION

NATURE

Propagation (travelling) of disturbance in elastic media:

- **gas (air)**
- **liquid (water)**
- **solid (crystal)**

CAUSE

Common effects:

- **each particle of medium „pass a message on” to its neighbour(s)**
- **transmition of energy of particles (oscillators) via coupling in elastic media.**

GENERAL CLASSIFICATION

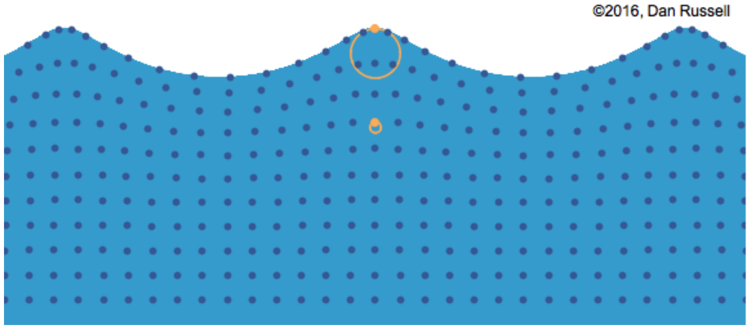
3 criteria: medium and source, boundary conditions, shape and angle

WAVES CLASSIFICATION

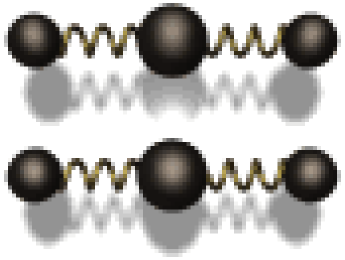
MEDIUM AND SOURCE

- **mechanical waves - a disturbance of elastic medium:**

water surface



string (phonon)



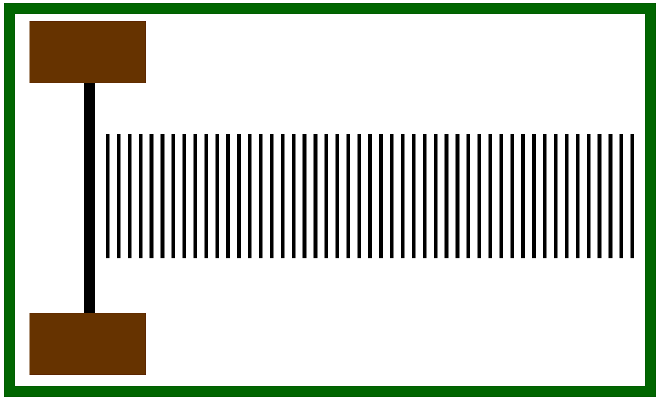
rubber hose

sound (air)

- **electromagnetic waves - a variation of intensity of electric E and magnetic H fields**

WAVES CLASSIFICATION

GEOMETRICAL BOUNDARY CONDITIONS OF PROPAGATION



- **plane waves**
  disturbance in one dimension -
  the wavefronts (surfaces of the constant phase) are parallel planes

- **spherical waves**
  disturbance in 3 dimensions
  from a point source
  - wavefronts concentric and spherical
  special case: waves at water surface from a point source -
  the concentric wavefronts.

WAVES CLASSIFICATION

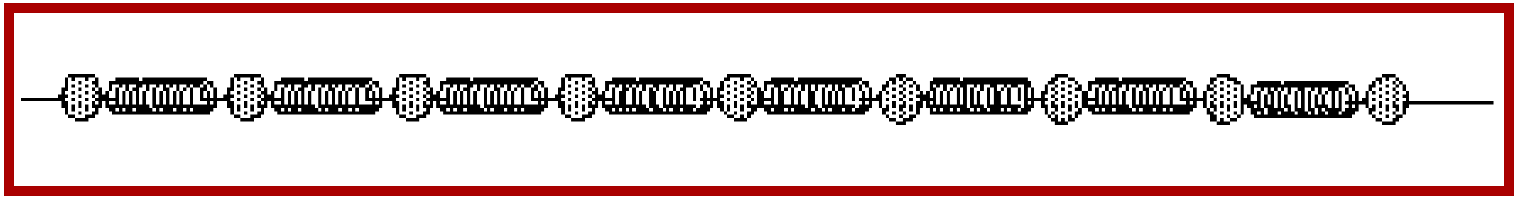
ANGLE: DISTURBANCE – DIRECTION OF PROPAGATION

- **longitudinal waves**

Oscillation (disturbance) II to propagation (velocity) - arbitrary media

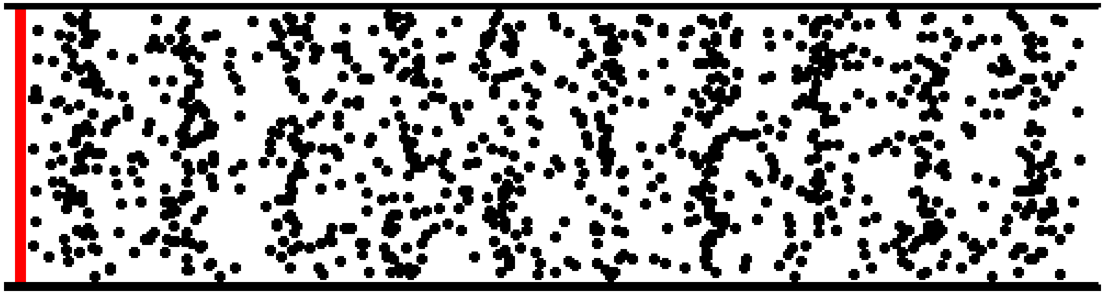
EXAMPLES:

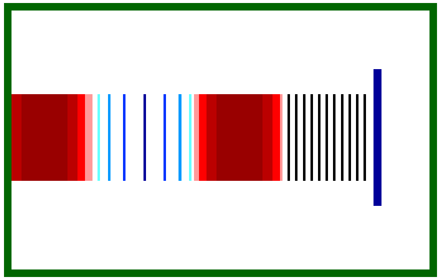
- **string**



Travelling of compression along its length

- **sound in air**





Compression and expansion – periodic changes of air density

WAVES CLASSIFICATION

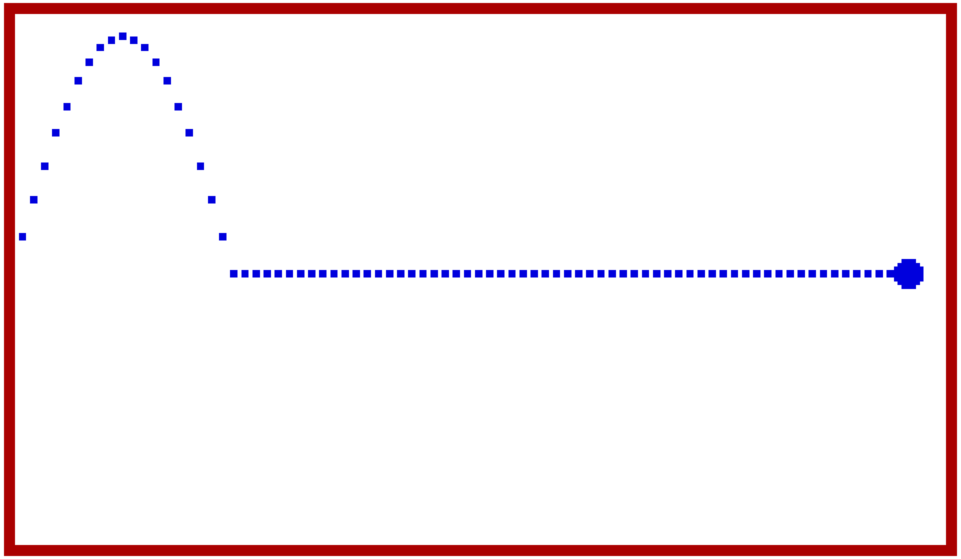
ANGLE: DISTURBANCE – DIRECTION OF PROPAGATION

- transverse waves

Oscillation (disturbance) ⊥ to propagation (velocity) - liquid, solid

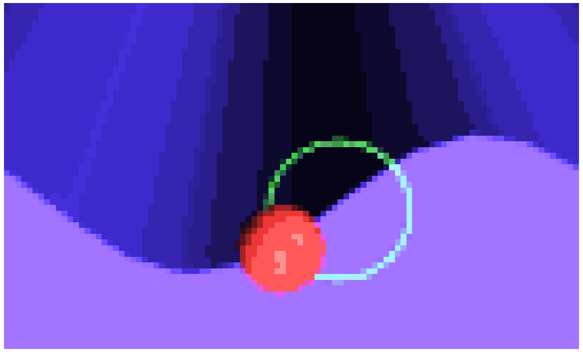
EXAMPLES:

- rubber hose



Travelling of disturbance along its length - harmonic wave at c. υ

- wave pulse at disturbed surface



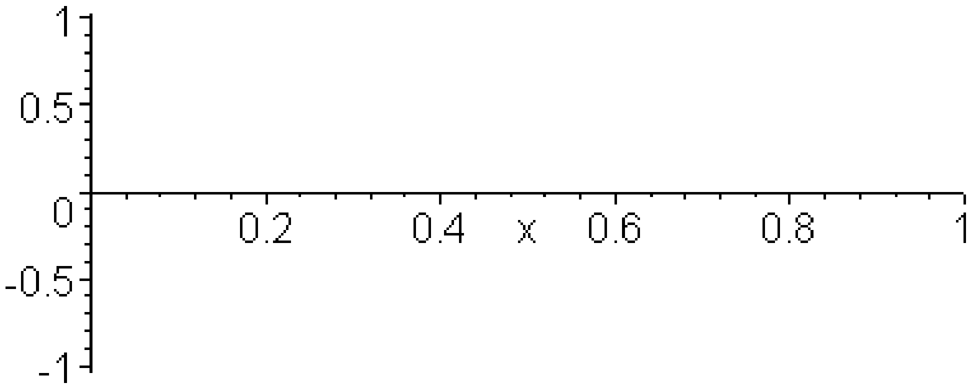
Travelling of disturbance along its length

- harmonic wave at c. υ

WAVE TRAVELLING

WAVE PROPAGATION IN ONE DIMENSION

Disturbance (wave) propagation along one direction *x*



For constant shape the wave function has a form

$$\Psi = \Psi(x', t) = \Psi(x \pm \upsilon \cdot t)$$

Twice differentiation with respect to *x* **and then with respect to** *t*

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{\upsilon^2} \cdot \frac{\partial^2 \Psi}{\partial t^2}$$

Differential equation of one dimensional wave

WAVE TRAVELLING

WAVE PROPAGATION IN ONE DIMENSION

Simplest case: plane wave propagation along one direction *x*
harmonic wave function

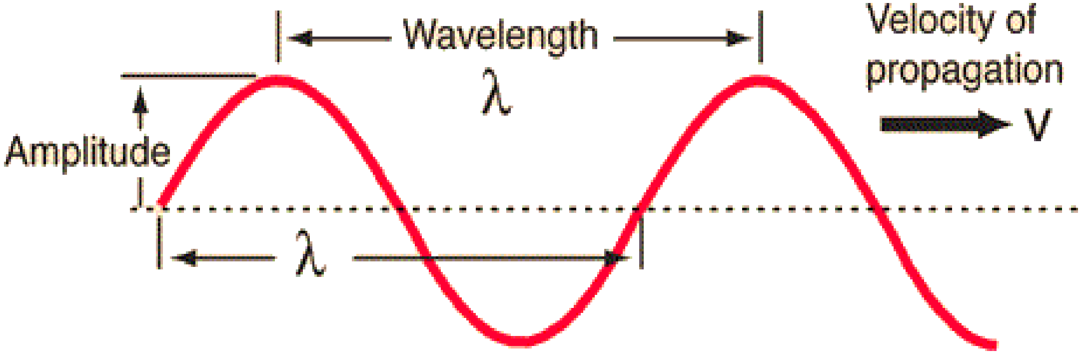
$$\Psi(x,t) = y(x,t) = A\cos(\omega t \pm \varphi) = A\cos(\omega t \pm kx)$$

where:

$k = \dfrac{2\pi}{\lambda}$ - **wave number**

$\lambda = \upsilon \cdot T$ - **wavelength**

$\upsilon = \dfrac{\lambda}{T} = \dfrac{\omega}{k}$ - **phase velocity**



Wave function of one dimensional wave

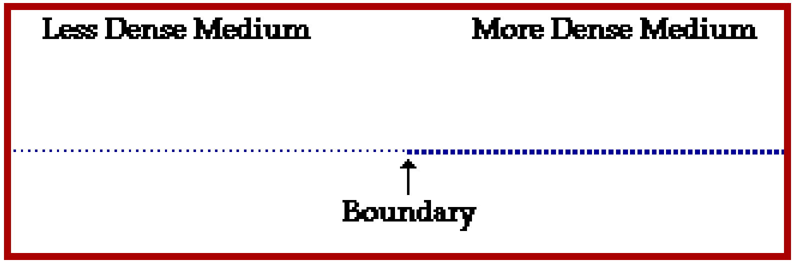
$$\Psi(x,t) = A\cos(\omega t \pm \varphi) = A\cos 2\pi\left(\frac{t}{T} \pm \frac{x}{\lambda}\right) = A\cos\frac{2\pi}{T}\left(t \pm \frac{x}{\upsilon}\right)$$

WAVE TRAVELLING

WAVE PROPAGATION AT INTERFACES

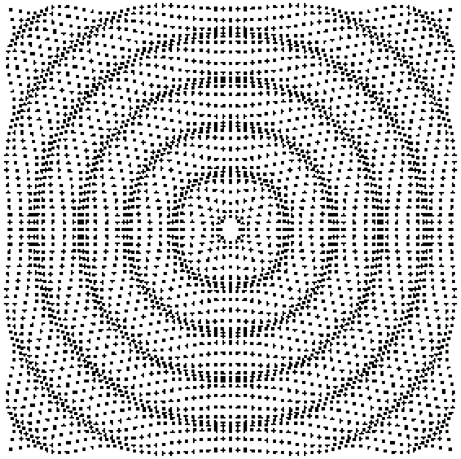
Wave travelling in a medium affected at interface:

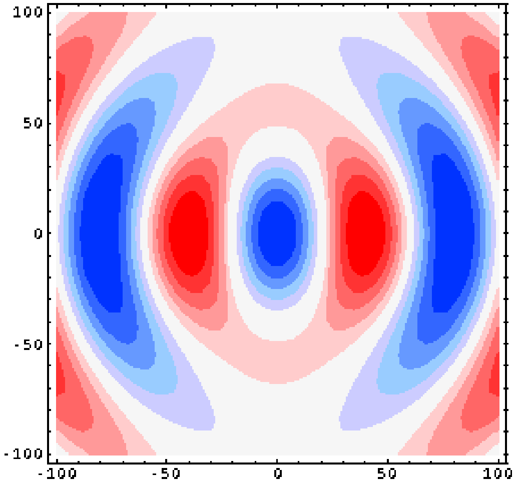
- **air – liquid**
- **air – solid**
- **liquid – solid**



Primary description - Huyghens' principle:
each point of wavefront considered as point source responsible for subsequent wave progress of wave

Wavefront is produced along envelope which is generated by elemental waves – shape of source determines the contour of wave



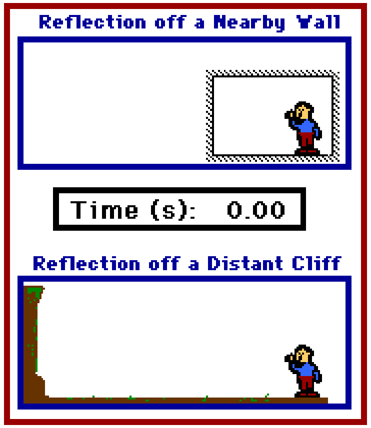


WAVE TRAVELLING

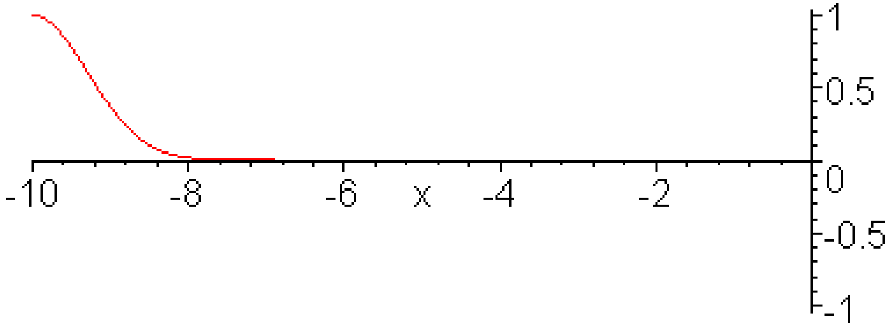
Reflection

- wave propagation $\perp$ to interface of different media of various velocity υ

reflection   partial reflection (transmision)



- wave propagation at angle to interface - variation of wave angles after reflection



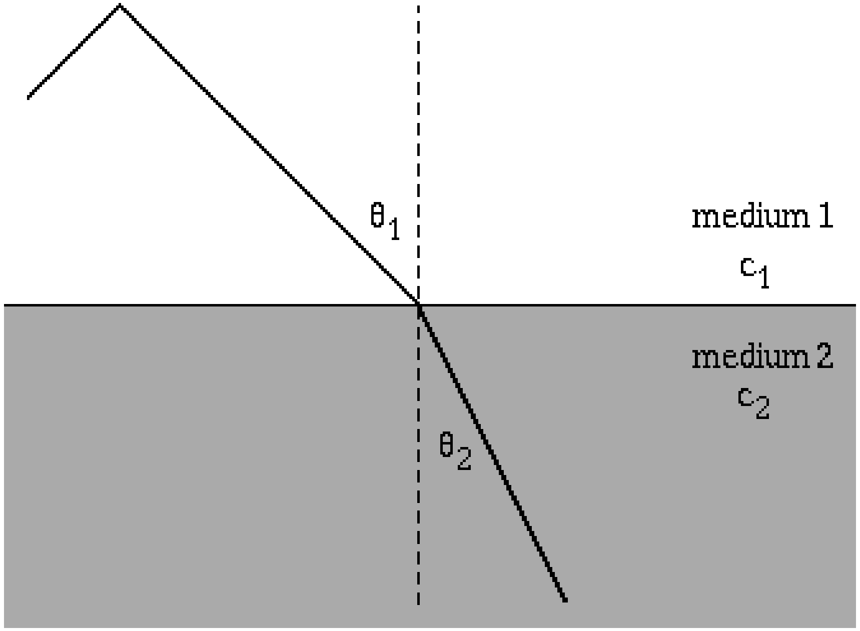
Law of wave reflection:

According to Huyghen’s principle - at interface angles of wave incidence and reflection are always equal with respect to normal.

WAVE TRAVELLING

Refraction

- wave propagation in different media exhibiting various velocity υ
  - effect of refraction at interface



Law of wave refraction:

According to Huyghen’s principle - angles of wave incidence and refraction (with respect to normal) determined by ratio of velocities in respected media

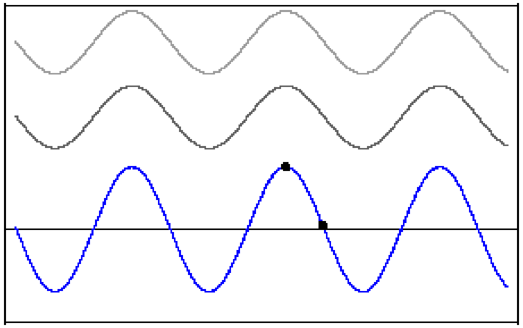
$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{c_1}{c_2}$$

WAVE TRAVELLING

Superposition

- two waves of different velocity and phase in same direction

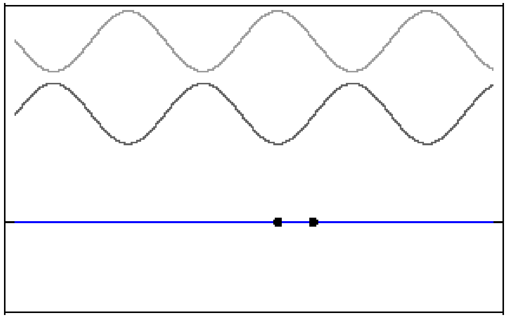
$$\Psi(x,t) = \cos\frac{2\pi}{T}\left(t+\frac{x}{\upsilon}\right)$$



- two waves of different velocity and phase in opposite direction

$$\Psi(x,t) = \cos\frac{2\pi}{T}\left(t+\frac{x}{\upsilon}\right)$$

$$\Psi(x,t) = \cos\frac{2\pi}{T}\left(t-\frac{x}{\upsilon}\right)$$



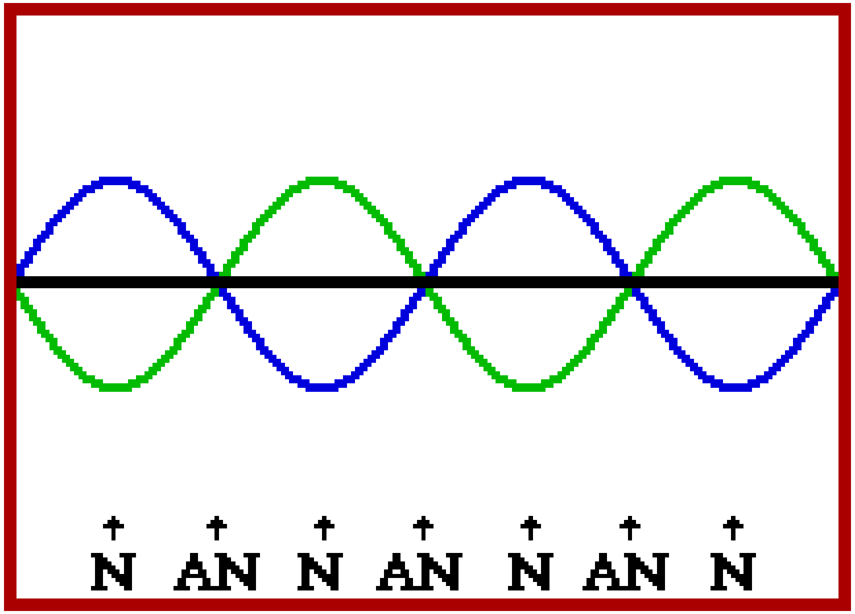
WAVE TRAVELLING

Superposition

- two waves of identical velocity and phase in opposite direction

$$\Psi(x,t) = \cos\frac{2\pi}{T}(t - \frac{x}{\upsilon})$$

$$\Psi(x,t) = \cos\frac{2\pi}{T}(t + \frac{x}{\upsilon})$$



Net wave function

$$\Psi(x,t) = 2\cos(\frac{2\pi}{\lambda}x)\cos\omega t$$

Two boundary conditions:

$$\cos(\frac{2\pi}{\lambda}x) = 0 \quad \text{thus} \quad \frac{2\pi}{\lambda}x = \pi(n + \frac{1}{2})$$

Node

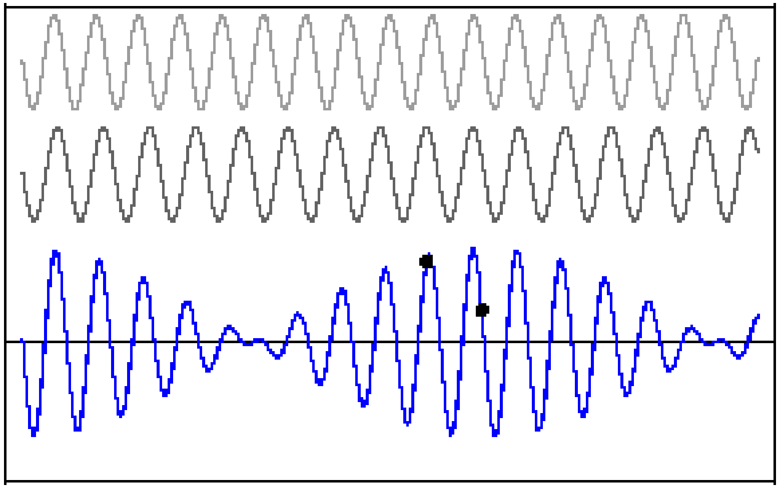
$$\cos(\frac{2\pi}{\lambda}x) = \pm 1 \quad \text{thus} \quad \frac{2\pi}{\lambda}x = \pi \cdot n$$

Anti-node

WAVE TRAVELLING

Superposition

- two waves of close velocity and phase in same direction



Envelope of particular waves modulating amplitude moving with group velocity generates localized wave pocket

Two different velocities:

- phase velocity inside

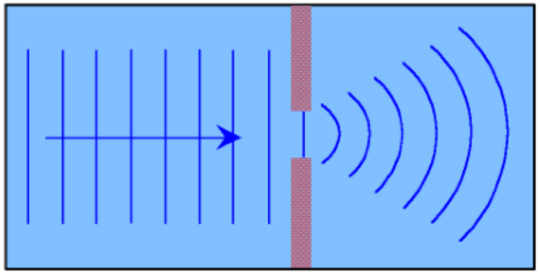
- group velocity of packet centre

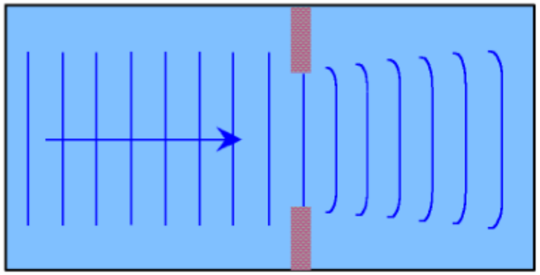
$$\upsilon_g = \lim_{\Delta k}\left(\frac{\Delta\omega}{\Delta k}\right) = \frac{d\omega}{dk} = \upsilon + k\frac{d\upsilon}{dk}$$

WAVE TRAVELLING

Diffraction and interference

- diffraction of one wave at slite(s) of different width

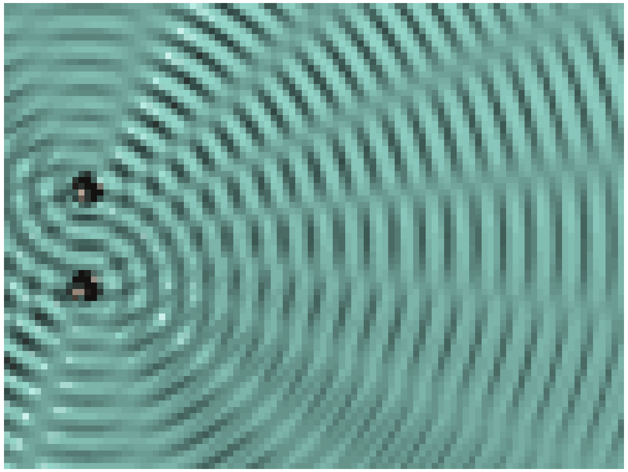




Different boundary effects:

- for narrow slit - generation of concentric waves
- for wide slit - generation of almost front plane waves

- diffraction and interference of two waves from close point slites



General tendency: overlapping of waves

Opposite boundary effects:

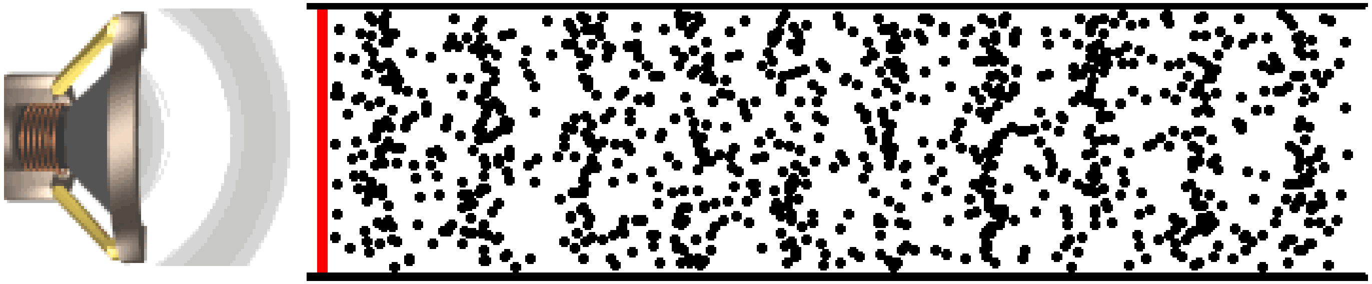
- phase coincidence - amplification
- phase anti-coincidence - weakening

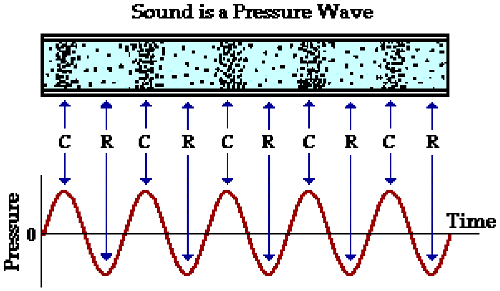
SOUND WAVES

NATURE

Elastic longitudinal waves travelling in disturbed air, liquid and solid

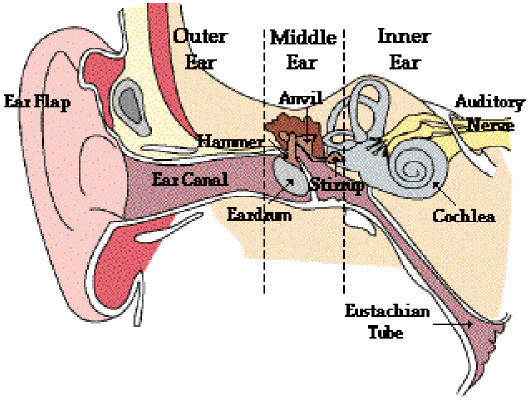
In air:
Air disturbance (compression and rarefaction) along direction of wave propagation - periodic changes of air density at constant velocity





Sound:

Sonic impression received by human ear generated by various sources in air as a result of its periodical disturbance.



SOUND WAVES

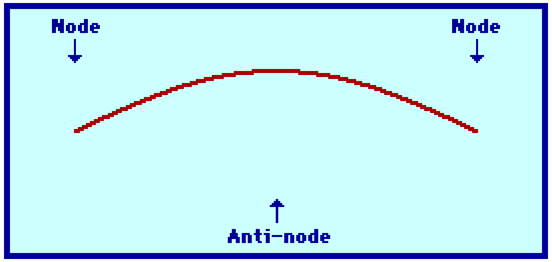
SOURCES

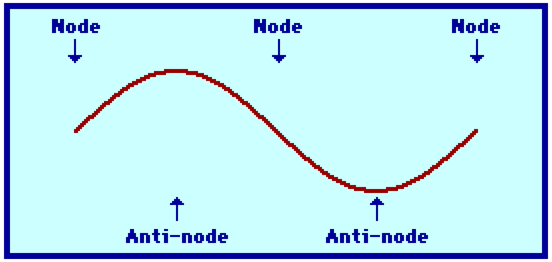
Every device causes periodic disturbance of air – general classification based on shape and dimension:

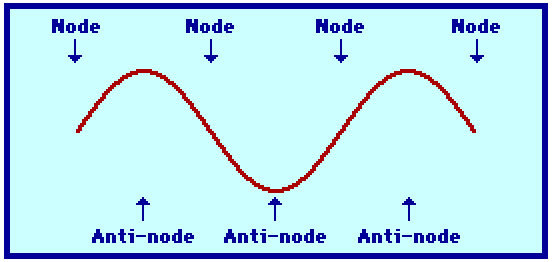
- **LINEAR**
  (one dimensional standing waves of nodal and antinodal points)

EXAMPLES:

- string







Transverse oscillations of disturbed air of frequency

$$f_n = \frac{n}{2} \cdot \frac{\upsilon}{l} = \frac{n}{2l}\sqrt{\frac{F}{s\rho}}$$

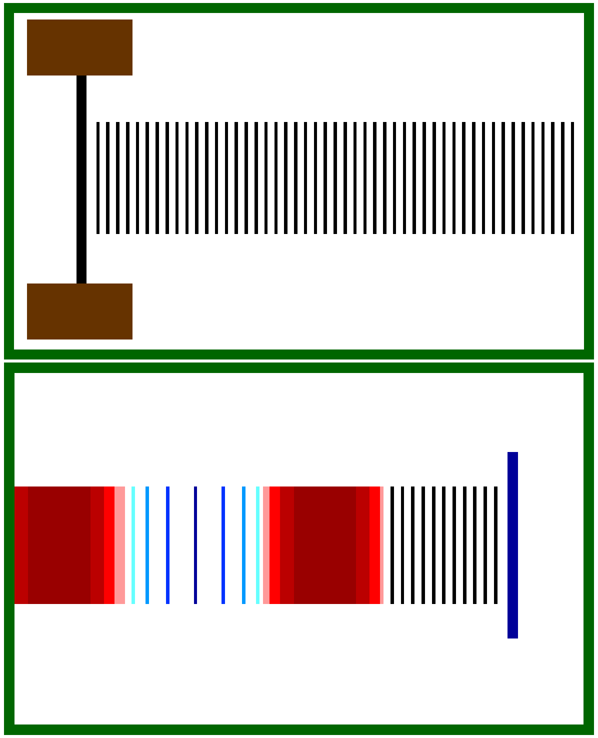
Along a string – total number of n half-waves:

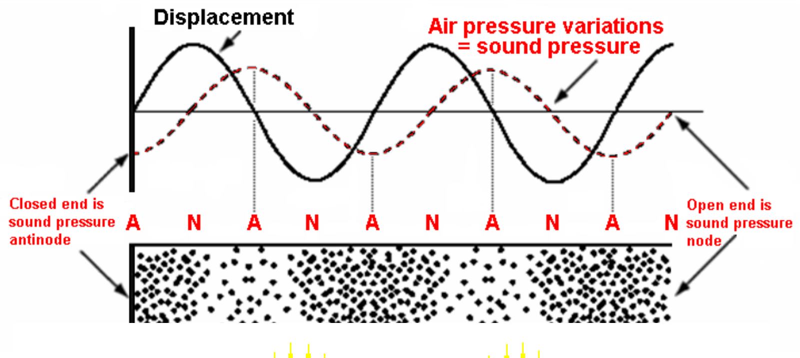
n=1 - base frequency; n=2,3,4 ... - harmonic frequency

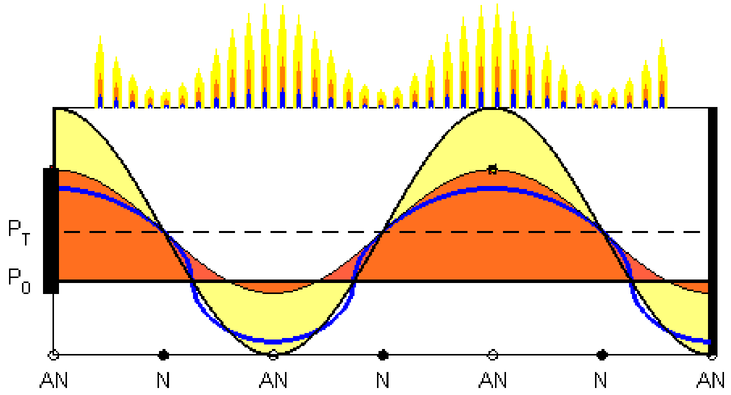
SOUND WAVES

- **LINEAR**

- air column







After motion of piston (membrane) inside tube shock compression of air -
- periodic changes of air density at constant sound velocity

$$\upsilon = \sqrt{\frac{B}{\rho}}$$

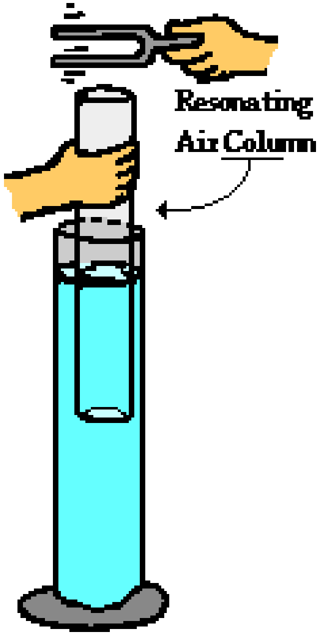
where: B - volumetric modulus of elasticity

SOUND WAVES

- **LINEAR**

- tuning fork

Fast vibrations of wings - oscillations of air column between wings - generation of harmonic wave of single base acoustic frequency: 435 Hz



Tuning fork forcing air coumn into resonance

Application:

Determination of sound velocity using standing wave resonance

$$\upsilon_a = \lambda_{sw} \cdot f_{tf} = 2d \cdot f_{tf}$$

For $f_{tf} = 435$ Hz

standing wave observed at air column length d = 0.38 m

- sound wave velocity in air $\upsilon = 330$ [m/s]

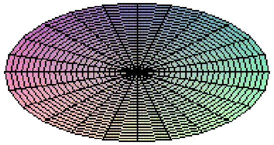
SOUND WAVES

- **PLANAR**

Two dimensional standing waves of nodal and antinodal lines strongly depends on geometry of supporting points)

EXAMPLES

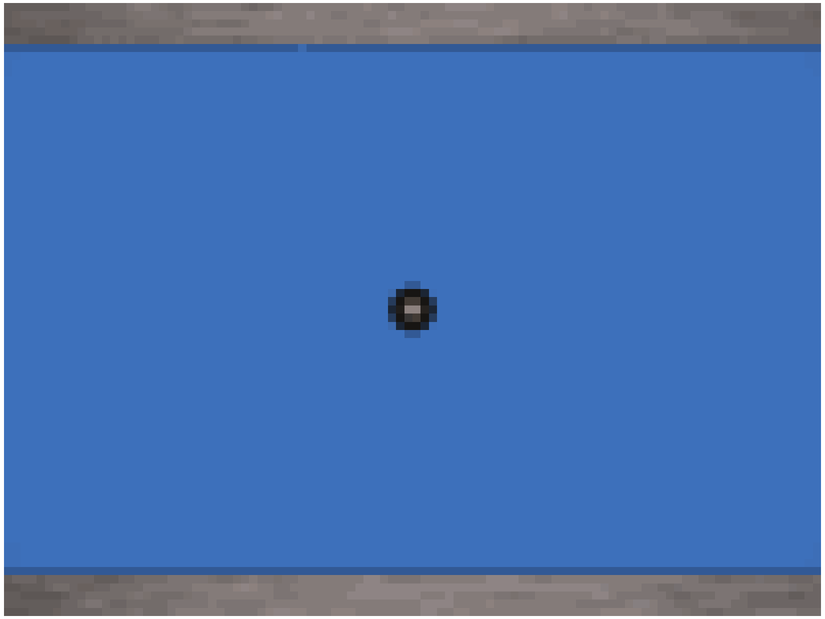
- circular membrane



Due to impact of membrane supported at different points – generation of first harmonic wave not being total multiple of base frequency

- plate

Due to impact of plate supported at different points – generation of Chladni figures – anharmonic oscillations of various shapes and base frequency dependent on plate's shape



SOUND WAVES

CHARACTERISTICS

Sound: periodic wave described by function of period T

$$\Psi = \Psi(t+T) = \sum_{n=0}^{\infty} C_n \cdot \cos(\omega_n t + \varphi_n)$$

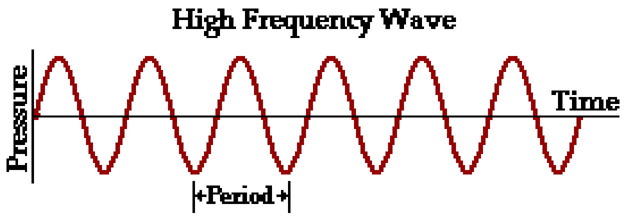
According to Fourier theorem - sum of convergent ininite series of subsequent harmonic oscillations of angular frequency

$$\omega_n = n \cdot \omega_o = n \cdot \frac{2\pi}{T}$$

where: n = 0, 1, 2, 3, ... n

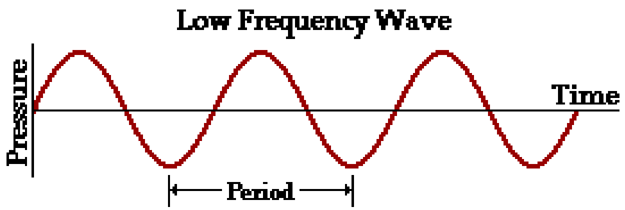
- multiple of base angular frequency ω_o

PARAMETERS



Frequency range: 16 ÷ 20000 Hz - acoustics
Main parameters (quantities) of audible sound:

- **frequency**
  received by human ear as height of sound

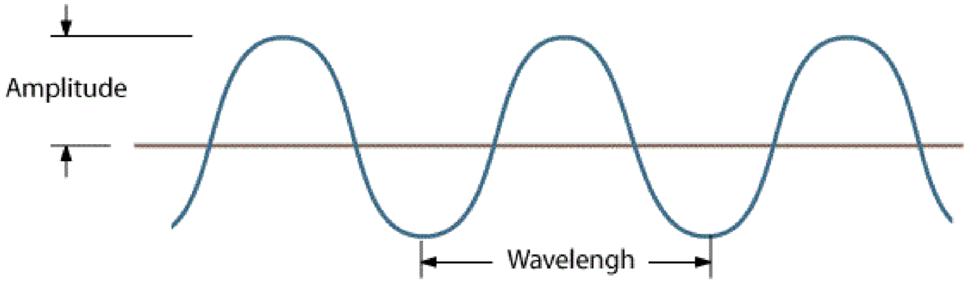


SOUND WAVES

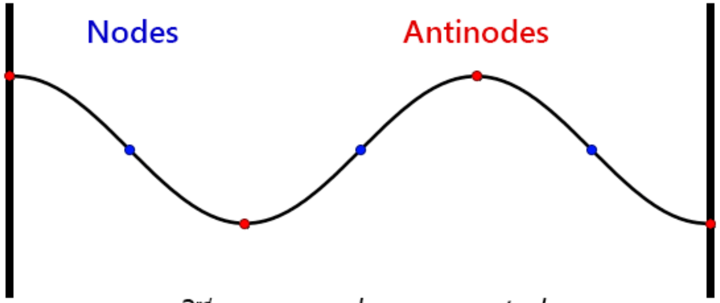
PARAMETERS

- **amplitude**

Registered as loudness (volume)



Sound Pressure of Axial Standing Waves in a Room



3rd room mode represented
In reflective rooms, walls are standing wave antinodes

- **spectrum**

In audible recognized as timbre

Amplitude dependence on frequency in 3 impressions:

- **tone:** **impression caused by periodic disturbance -**
  line spectrum at chosen frequency - tuning fork
- **sound:** **impression caused by anharmonic disturbance -**
  line spectrum of various amplitudes and frequency -
- **murmur:** **impression caused by aperiodic continuos disturbance**
  strong murmur of increasing amplitude: rumble, crash

SOUND WAVES

INTENSITY

Sound as impression - different sound intensity:

- **physical (absolute) intensity**

Average energy of sound travelling at velocity υ via section S in time t

$$I = \frac{E}{S \cdot t}$$

Average energy of sound wave

$$E = \frac{1}{2} mA^2\omega^2 = \frac{1}{2}(\rho V)A^2(2\pi f)^2 = 2\pi^2\rho \cdot A^2 \cdot f^2 \cdot S \cdot \upsilon \cdot t$$

Physical (absolute) sound intensity

$$I = \frac{E}{S \cdot t} = 2\pi^2 \cdot \rho \cdot f^2 \cdot A^2 \cdot \upsilon$$

Unit: absolute [W/m^2]

SOUND WAVES

INTENSITY

- subjective intensity

Average energy of sound received by human ear as an impresion

Because of different individual sensitivity of physical sound intensity – relative parameter in logarythmic scale – sound impression level - Weber-Ferchner law

$$\beta = 10\,\log\frac{I}{I_o} = 10\,\log\frac{I}{10^{-12}}$$

where: I_o - audible threshold of physical intensity for tone at 1kHz

Relative unit: decibel [dB]

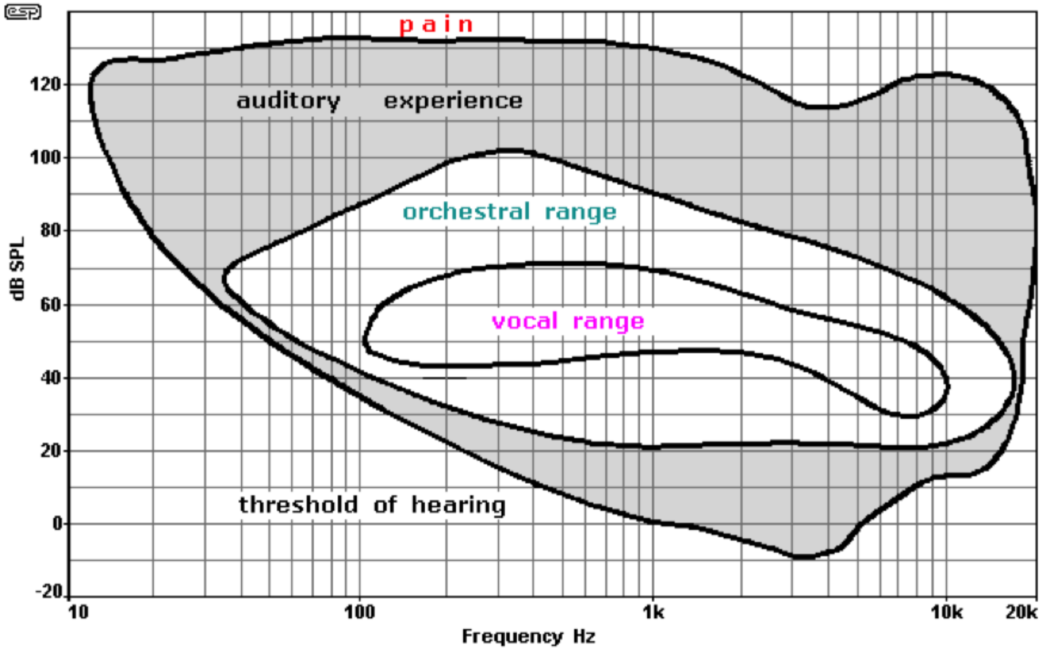
Example: for $I = 1000\ I_o$ - sound impression level β = log1000 = 30 [dB]

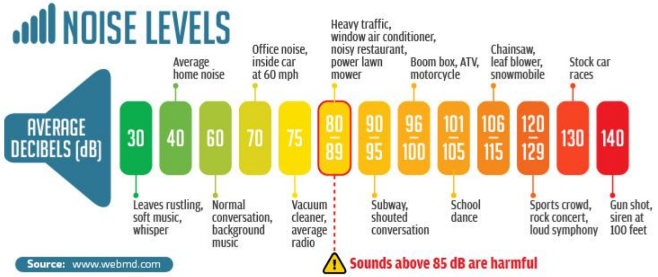
SOUND WAVES

INTENSITY

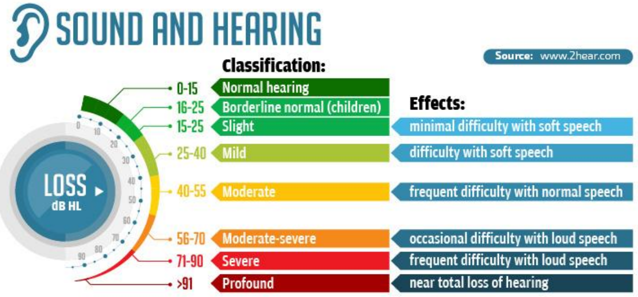
- **subjective intensity**

Because of strong dependence of sound received by human ear on frequency f - additional relative parameter: perceived noise level





Source: www.webmd.com



Source: www.2hear.com

	Classification:	Effects:
0-15	Normal hearing	
16-25	Borderline normal (children)	
15-25	Slight	minimal difficulty with soft speech
25-40	Mild	difficulty with soft speech
40-55	Moderate	frequent difficulty with normal speech
56-70	Moderate-severe	occasional difficulty with loud speech
71-90	Severe	frequent difficulty with loud speech
>91	Profound	near total loss of hearing

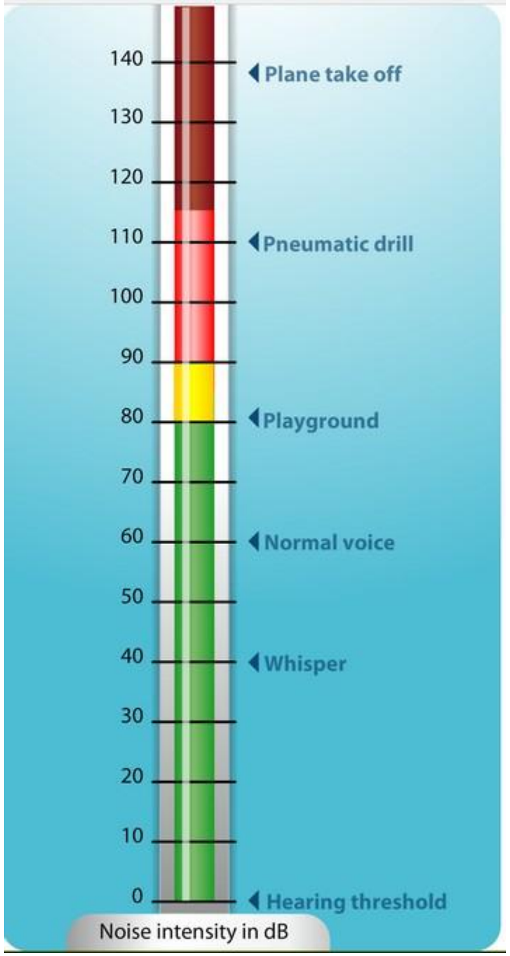
Relative unit: phon - sound impression level [dB] of tone at f = 1 [kHz]

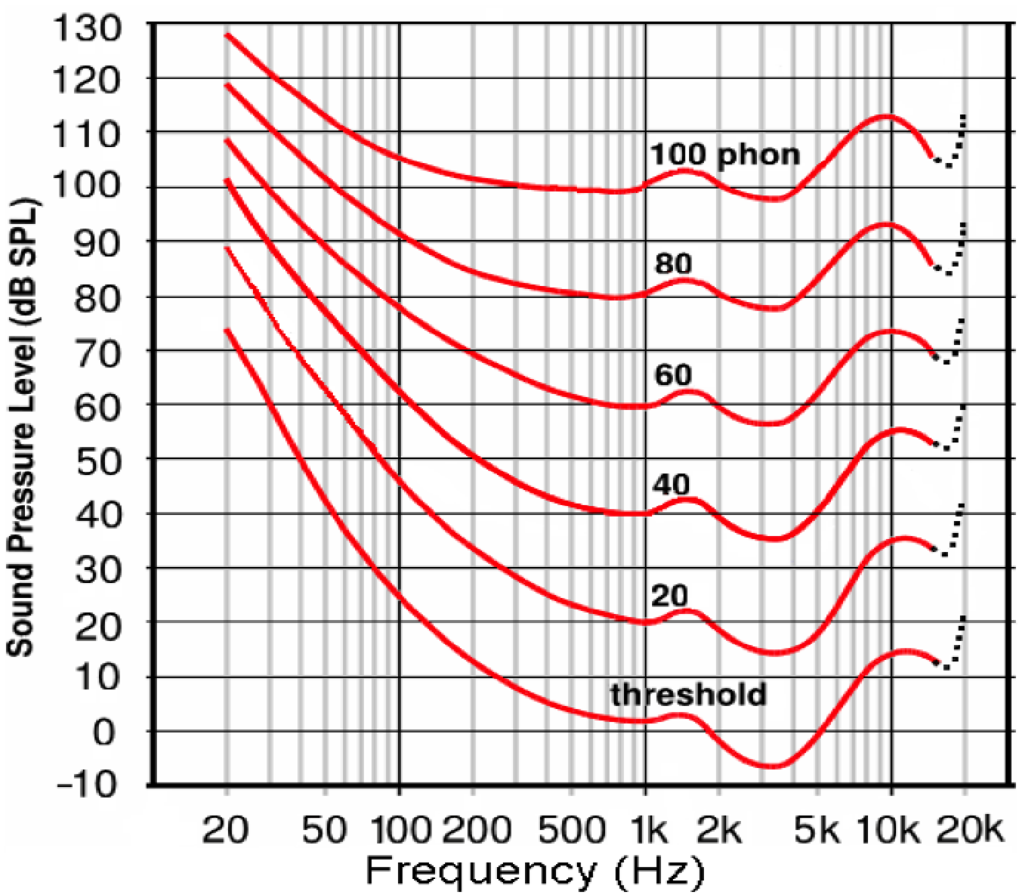
SOUND WAVES

INTENSITY

- subjective intensity

Comparison of perceived noise level of various sound sources [phons]





Equal-loudness Curves ISO 226:2003

SOUND WAVES

DOPPLER EFFECT

Change of length (frequency) of sound wave received by observer during variation of distance: source-observer - two boundary cases: example

- **sound source in motion**

Generation of sound of velocity, length and frequence - two possibilities: observer in rest approaches or dismisses

sound received by observer shorter in λ - higher in *f*

observer:

$$f' = \frac{\upsilon}{\lambda'} = \frac{\upsilon}{(\upsilon - \upsilon_s)/f} = f\left(\frac{\upsilon}{\upsilon - \upsilon_s}\right)$$

for motion in opposite direction – opposite manner and relation

$$f' = \frac{\upsilon}{\lambda'} = \frac{\upsilon}{(\upsilon + \upsilon_s)/f} = f\left(\frac{\upsilon}{\upsilon + \upsilon_s}\right)$$